

## 201-103-RE - Calculus 1

### WORKSHEET: CURVE SKETCHING

#### General Guidelines

(1) **domain of  $f(x)$**

(2) **intercepts**

(3) **asymptotes**

(a) horizontal asymptotes

$$\lim_{x \rightarrow \infty} f(x) \quad \text{and} \quad \lim_{x \rightarrow -\infty} f(x)$$

(b) vertical asymptotes

$$\lim_{x \rightarrow a^-} f(x) = \pm\infty \quad \text{and} \quad \lim_{x \rightarrow a^+} f(x) = \pm\infty$$

(4) **critical points of  $f(x)$** : when  $f'(x) = \text{DNE}$  or  $f'(x) = 0$

(5) **intervals of increase/decrease**: given by the sign of  $f'(x)$

(6) **local minimums and maximums**: First or Second Derivative Test

(7) **concavity and points of inflection**: given by the sign of  $f''(x)$

(8) **important points**: list all the interesting points in a table with their  $y$ -values.

(9) **SKETCH THE GRAPH!**

#### Polynomial functions

(1)  $f(x) = x^3 + x^2 - x + 2$

(2)  $f(x) = x^5 - 5x + 1$

(3)  $f(x) = (x + 4)^3(x - 2)$

(4)  $f(x) = 2 - x - x^3$

(5)  $f(x) = 3x^4 - 6x^2$

(6)  $f(x) = x^3 - 3x^2 + 3$

(7)  $f(x) = 2x^4 - x^2$

#### Rational functions

(1)  $f(x) = \frac{x^2 + 4}{x^2 - 4}$

(2)  $f(x) = \frac{2x - x^2}{x^2 - 2x + 1}$

(3)  $f(x) = \frac{x}{(x + 1)^2}$

(4)  $f(x) = \frac{3(x^2 + 1)}{x^2 - 9}$

(5)  $f(x) = \frac{9x}{(3x + 1)^2}$

#### Others

(1)  $f(x) = \sqrt[3]{x}(x + 8)$

(2)  $f(x) = \frac{e^x}{x}$

### Derivatives for the rational functions

$$(1) \quad f(x) = \frac{x^2 + 4}{x^2 - 4}, \quad f'(x) = \frac{-16x}{(x^2 - 4)^2}, \quad f''(x) = \frac{16(3x^2 + 4)}{(x^2 - 4)^3}$$

$$(2) \quad f(x) = \frac{2x - x^2}{x^2 - 2x + 1}, \quad f'(x) = \frac{-2}{(x - 1)^3}, \quad f''(x) = \frac{6}{(x - 1)^4}$$

$$(3) \quad f(x) = \frac{x}{(x + 1)^2}, \quad f'(x) = \frac{1 - x}{(x + 1)^3}, \quad f''(x) = \frac{2(x - 2)}{(x + 1)^4}$$

$$(4) \quad f(x) = \frac{3(x^2 + 1)}{x^2 - 9}, \quad f'(x) = \frac{-60x}{(x^2 - 9)^2}, \quad f''(x) = \frac{180(x^2 + 3)}{(x^2 - 9)^3}$$

$$(5) \quad f(x) = \frac{9x}{(3x + 1)^2}, \quad f'(x) = \frac{9(1 - 3x)}{(3x + 1)^3}, \quad f''(x) = \frac{54(3x - 2)}{(3x + 1)^4}$$

### Derivatives for the other functions

$$(1) \quad f(x) = x^{1/3}(x + 8), \quad f'(x) = \frac{4(x + 2)}{3x^{2/3}}, \quad f''(x) = \frac{4(x - 4)}{9x^{5/3}}$$

$$(2) \quad f(x) = \frac{e^x}{x}, \quad f'(x) = \frac{e^x(x - 1)}{x^2}, \quad f''(x) = \frac{e^x(x^2 - 2x + 2)}{x^2}$$