Calculus I
Curve Sketching

Polynomials

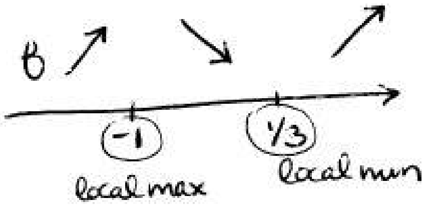
(1) domain = $\mathbb{R}$

y-intercept: $(0, 2)$

$f'(x) = (3x-1)(x+1)$

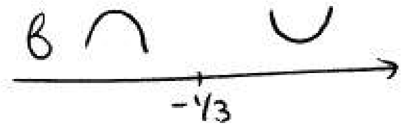
$f''(x) = 2(3x+1)$

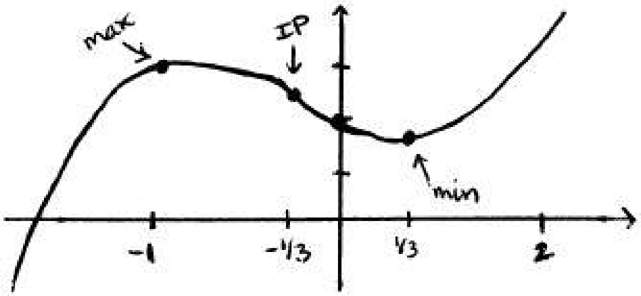
critical points: $x = -1, \frac{1}{3}$



PIPS: $x = -\frac{1}{3}$ ← inflection point

x	f
0	2
-1	3
1/3	1.8
-1/3	2.4





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(2) $f(x) = x^5 - 5x + 1$

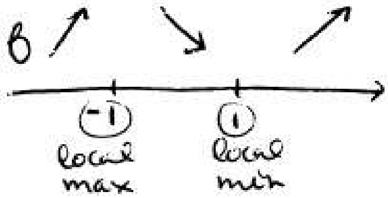
domain = $\mathbb{R}$

y-intercept $(0, 1)$

$f'(x) = 5(x^4 - 1)$

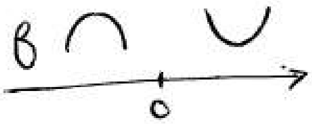
$f''(x) = 20x^3$

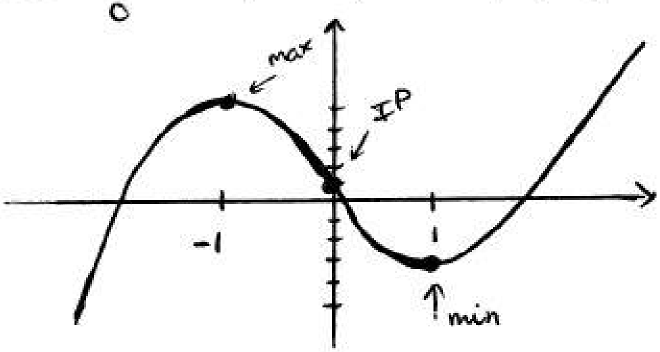
critical points. $x = \pm 1$



PIPS. $x = 0$ ← inflection

x	f(x)
0	1
-1	5
1	-3





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(3) $f(x) = (x+4)^3(x-2)$

domain = $\mathbb{R}$

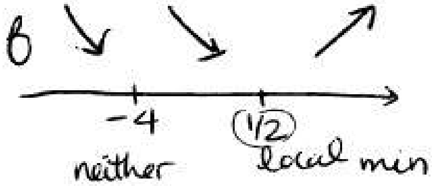
y-intercept: $(0, -128)$

x-intercept: $(-4, 0)$, $(2, 0)$

$f'(x) = 2(x+4)^2(2x-1)$

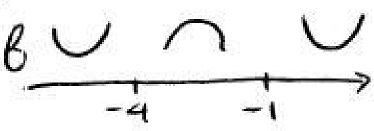
$f''(x) = 12(x+4)(x+1)$

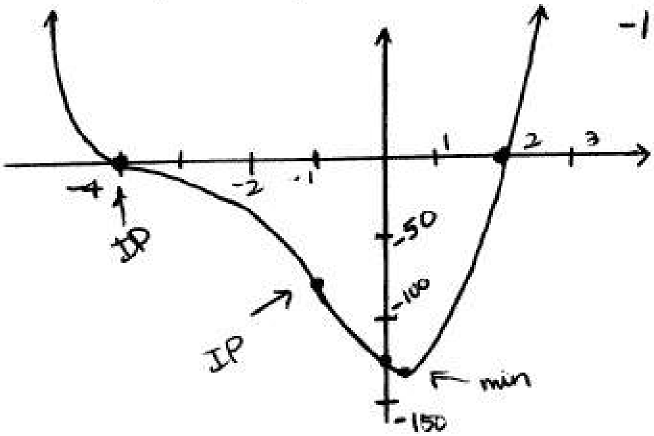
critical pts: $x = -4, \frac{1}{2}$



PIPS: $x = -4$, $x = -1$

x	f(x)
0	-128
-4	0
2	0
1/2	-137
-1	-81





(4) $f(x) = 2 - x - x^3$

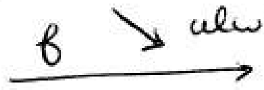
domain = $\mathbb{R}$

y-intercept = (0,2)

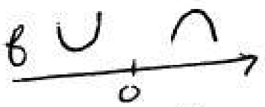
$f'(x) = -(3x^2+1)$

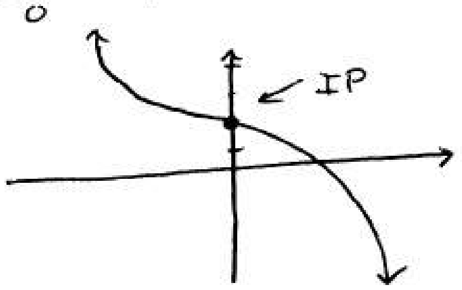
$f''(x) = -6x$

critical points: $x = \underline{\text{none}}$



PIPs: $x=0$ ← inflection





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(5) $f(x) = 3x^2(x^2-2)$

domain = $\mathbb{R}$

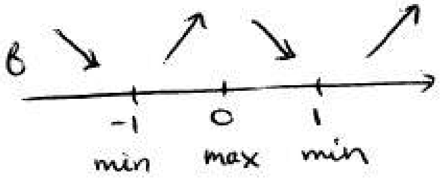
y-intercept = (0,0)

x-intercepts = (0,0), (±1.4, 0)

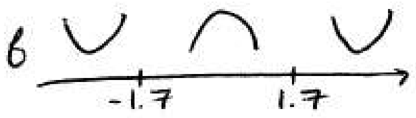
$f'(x) = 12x(x^2-1)$

$f''(x) = 12(3x^2-1)$

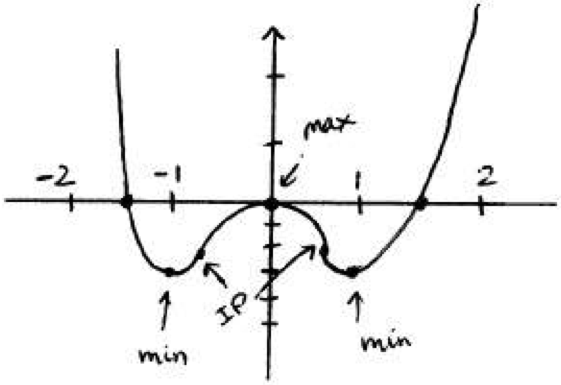
critical points: $x = 0, \pm 1$



PIPs: $x = \pm 1.7$ ← inflection



x	y
0	0
±1.4	0
±1	-3
±0.56	-27



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(6) $f(x) = x^3 - 3x^2 + 3$

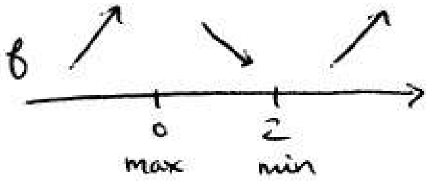
domain = $\mathbb{R}$

y-intercept = (0,3)

$f'(x) = 3x(x-2)$

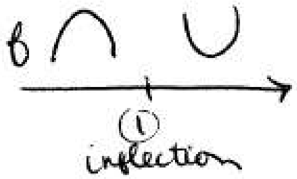
$f''(x) = 6(x-1)$

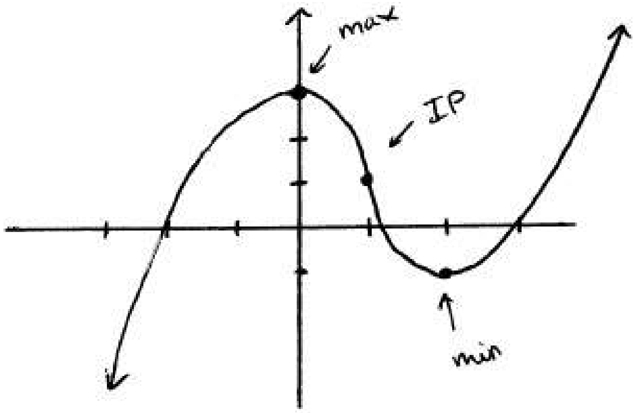
critical points: $x=0, x=2$



x	f(x)
0	3
2	-1
1	1

PIP: $x = 1$





(7) $f(x) = 2x^4 - x^2 = x^2(2x^2-1)$

domain = $\mathbb{R}$

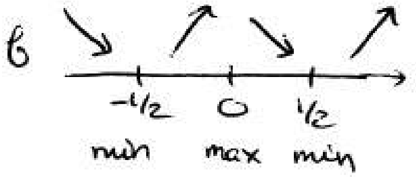
y-intercept : $(0,0)$

x-intercepts: $(0,0)$, $(\pm 0.7, 0)$

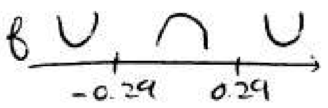
$f'(x) = 2x(4x^2-1)$

$f''(x) = 2(12x^2-1)$

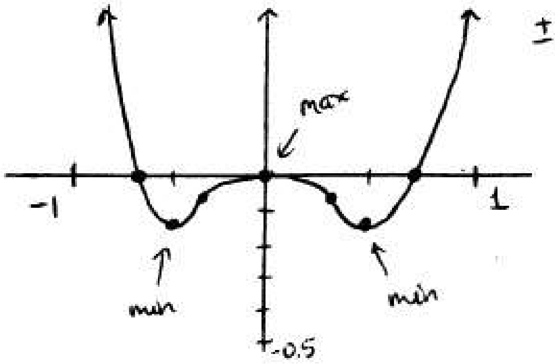
critical points $x=0, \pm 1/2$



PIPs : $x = \pm 0.29$



x	y
0	0
±0.7	0
±1/2	-0.125
±0.29	-0.07



Rational functions

(1) $f(x) = \dfrac{x^2+4}{x^2-4}$

domain: $\mathbb{R}\setminus\{\pm 2\}$

y-intercept: $(0,-1)$

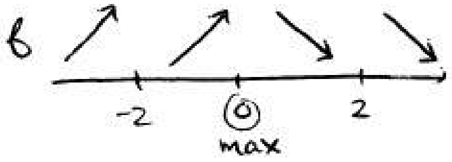
x-intercept: <u>none</u>

v.a. $x = \pm 2$

h.a. $y = 1$

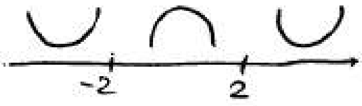
$f'(x) = \dfrac{-16x}{(x^2-4)^2}$

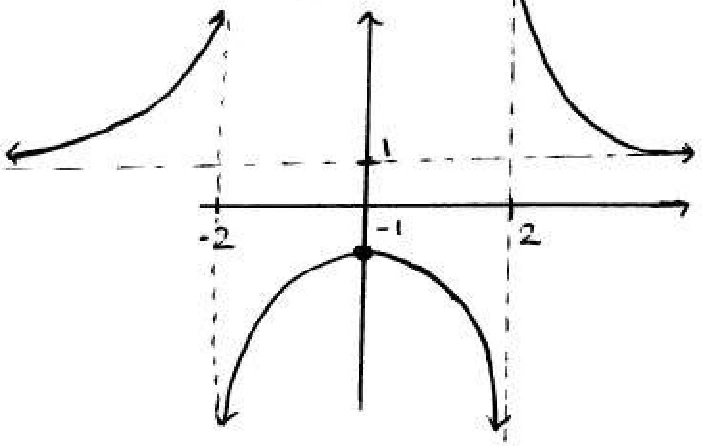
critical points $x=0, \pm\cancel{2}$



$f''(x) = \dfrac{16(3x^2+4)}{(x^2-4)^3}$

PIPs : $x = \pm\cancel{2}$ ← <u>not</u> inflection points!





(2) $f(x) = \dfrac{x(2-x)}{(x-1)^2}$

domain = $\mathbb{R}\setminus\{1\}$

y-intercept : $(0,0)$

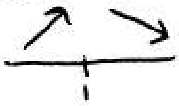
x-intercepts: $(0,0)$, $(2,0)$

v.a. $x=1$

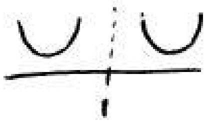
h.a. $y=-1$

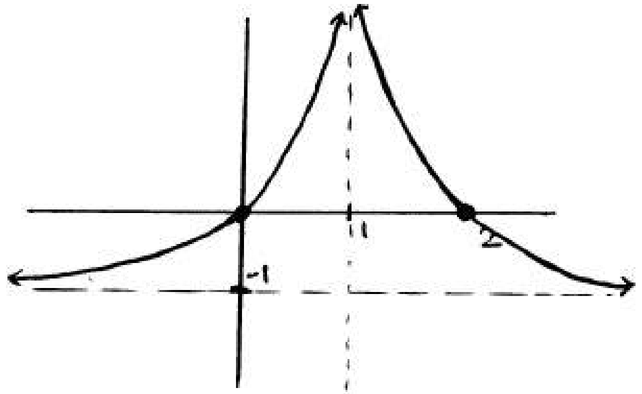
$f'(x) = \dfrac{-2}{(x-1)^3}$

critical points: ~~x=1~~



$f''(x) = \dfrac{6}{(x-1)^4}$ PIP: ~~x=1~~





(3) $f(x) = \frac{x}{(x+1)^2}$

domain = $\mathbb{R}\setminus\{-1\}$

y-intercept: (0,0)

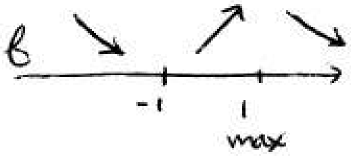
x-intercept: (0,0)

v.a. $x = -1$

h.a. $y = 0$

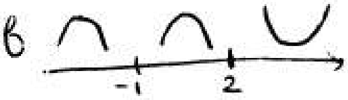
$f'(x) = \frac{1-x}{(x+1)^3}$

critical points: $x = \cancel{-1}, 1$

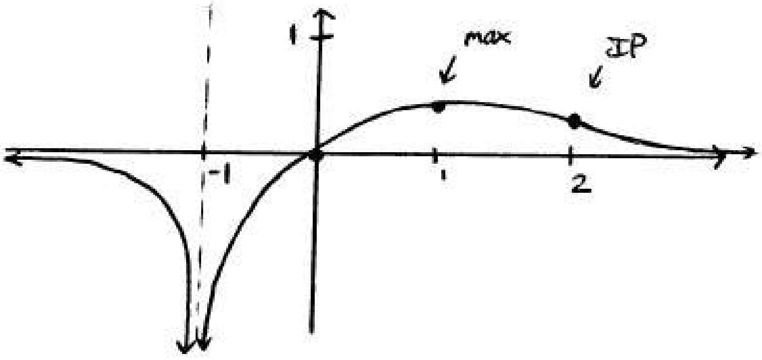


$f''(x) = \frac{2(x-2)}{(x+1)^4}$

PIPs: $x = \cancel{-1}, \textcircled{2} \rightarrow$ IP



x	f(x)
0	0
-1	v.a.
1	0.25
2	0.22



(4) $f(x) = \frac{3(x^2+1)}{x^2-9}$

domain = $\mathbb{R}\setminus\{\pm3\}$

y-intercept: $(0, -1/3)$

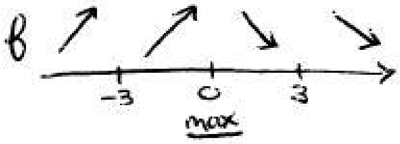
x-intercept: none

v.a.: $x = \pm3$

h.a.: $y = 3$

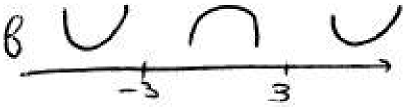
$f'(x) = \frac{-60x}{(x^2-9)^2}$

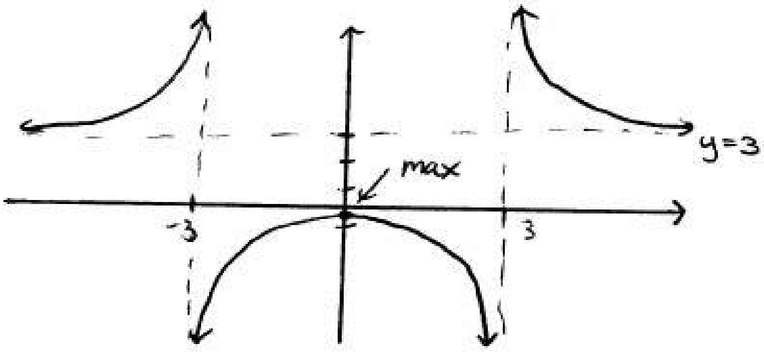
critical points: $x = 0, \cancel{\pm3}$



$f''(x) = \frac{180(x^2+3)}{(x^2-9)^3}$ PIPs: $x = \cancel{\pm3}$

(no inflection pts)





(5) $f(x) = \frac{9x}{(3x+1)^2}$

domain = $\mathbb{R}\setminus\{-1/3\}$

y-intercept: (0,0)

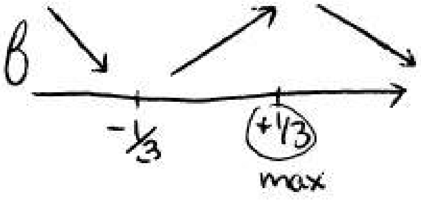
x-intercept: (0,0)

v.a.: $x = -1/3$

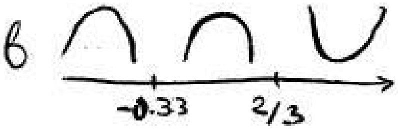
h.a.: $y = 0$

$f'(x) = \frac{9(1-3x)}{(3x+1)^3}$

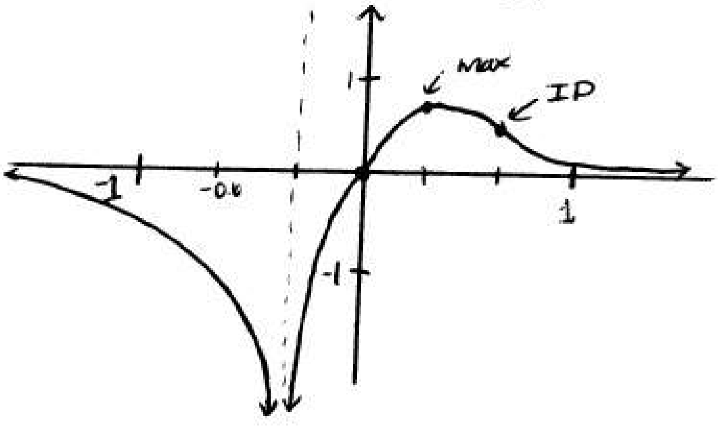
critical points: $x = +\frac{1}{3}, \cancel{-1/3}$



$f''(x) = \frac{54(3x-2)}{(3x+1)^4}$ PIPs: $x = \cancel{-1/3}, \textcircled{2/3}$ inflection point.



x	f(x)
0	0
1/3	0.75
2/3	2/3 = 0.67



Others

(1) $f(x) = x^{1/3}(x+8)$

domain = $\mathbb{R}$

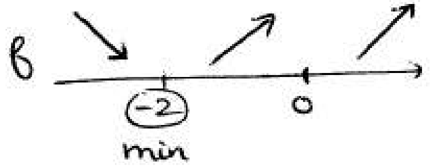
y-intercept: $(0,0)$

x-intercept: $(0,0), (-8,0)$

$\lim_{x\to\infty} f(x) = \infty \quad \lim_{x\to-\infty} f(x) = \infty$

$$f'(x) = \frac{4(x+2)}{3x^{2/3}}$$

critical points: $x=0,-2$

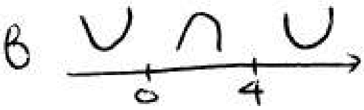


$$f''(x) = \frac{4(x-4)}{9x^{5/3}}$$

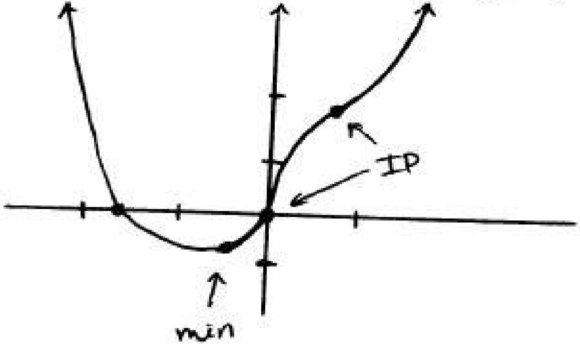
PIPs: $x=0,4$

↑↑

both inflection points



x	0	-8	-2	4
$f(x)$	0	0	-7.6	19.05



(2) ⇒ $f(x) = \dfrac{e^x}{x}$

$\boxed{\text{V.a. } x=0}$

domain = $\mathbb{R}\setminus\{0\}$

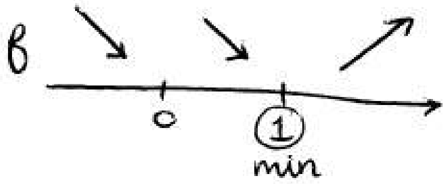
y-intercept: none (v.a.)

x-intercept: none.

$\lim_{x\to\infty} = \infty$, $\lim_{x\to-\infty} = 0$

$$f'(x) = \frac{e^x(x-1)}{x^2}$$

critical points: $x=\cancel{0}, 1$



$$f''(x) = \frac{e^x(x^2-2x+2)}{x^2}$$

no roots

PIPs: $x=\cancel{0}$

not inflection