

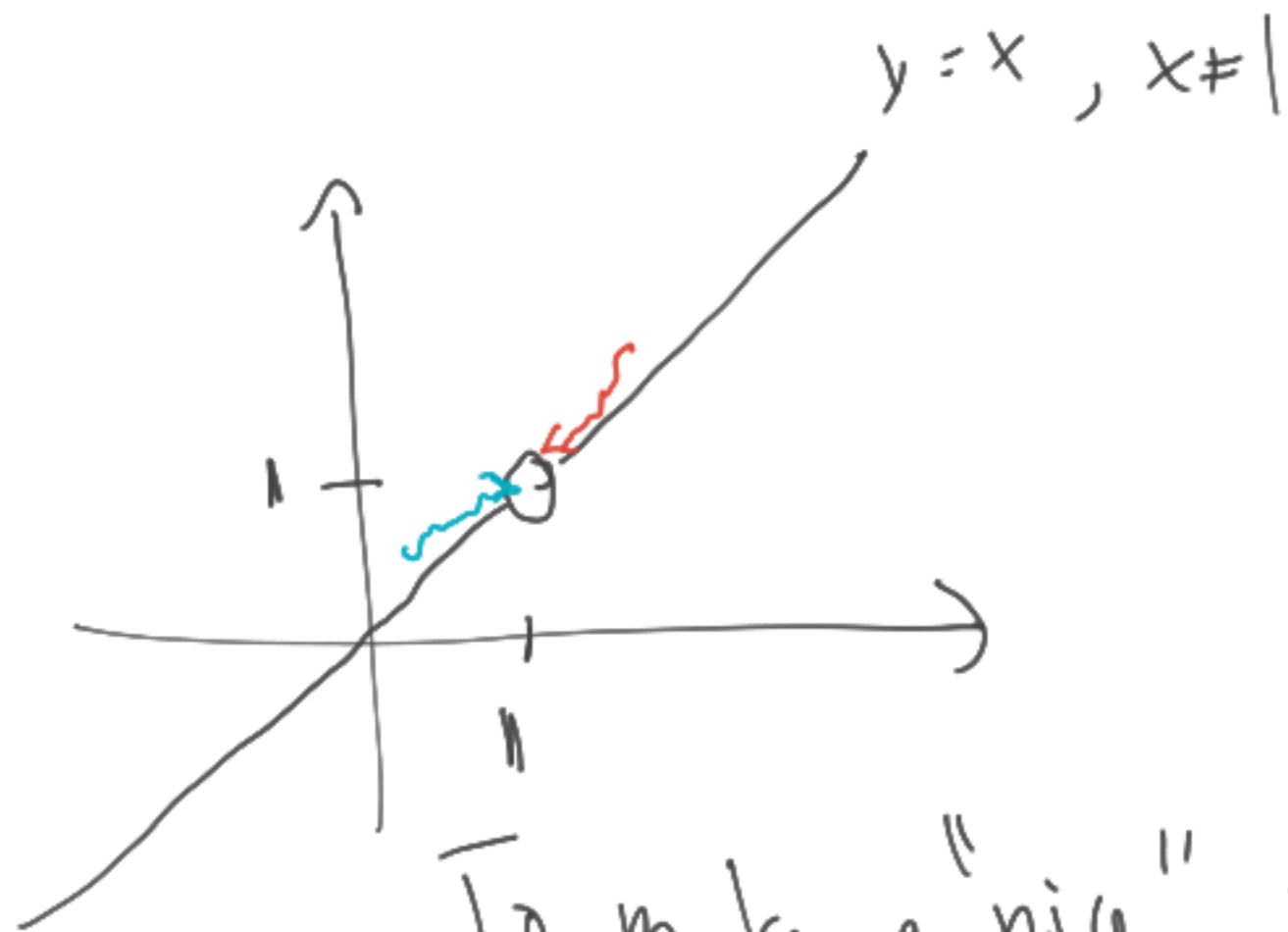
Limits

$$f(x) = x, x \neq 1$$
$$y = x, x \neq 1$$

(complete limit)

$$\lim_{x \rightarrow 1} f(x) = 1$$

since both one-sided limits
are equal to 1.



$f(1)$ DNE

To make a "nice" fn, it would be
nice if $f(1)$ was 1.

$$\lim_{x \rightarrow 1^-} f(x) = 1$$

left-sided limit

$$\lim_{x \rightarrow 1^+} f(x) = 1$$

right-sided limit

Ex: $y = \frac{x^2 - 1}{x - 1} = \frac{(x-1)(x+1)}{x-1} = \frac{\cancel{x-1}}{\cancel{x-1}} (x+1) = x+1, x-1 \neq 0$

cts everywhere except at $x=1$

$y(1)$ DNE
 but $\lim_{x \rightarrow 1} y = 2$

$x+1, x \neq 1$

cannot divide by zero

Continuous means "you don't have to let go of your pen"

Consider $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1} = \lim_{x \rightarrow 1} \frac{(x-1)(x+1)}{\cancel{(x-1)}} = \lim_{x \rightarrow 1} (x+1) = 1+1 = 2$

$x \rightarrow 1$
close to 1
(but not at 1)

\uparrow
denom $\neq 0$
close to zero