Fact:

$$\lim_{x \to a} p(x) = p(a) \quad \text{for polynomials } p(x)$$

$$a x^n + a_{n-1} x^{n-1} + \cdots a_1 x + a_0$$

e.g. $5x^3 - x + 1$

$x^2 + 7$

$x - x^8$

$$\lim_{x \to a} \frac{p(x)}{q(x)} = \frac{p(a)}{q(a)} \quad \text{if } q(a) \neq 0$$

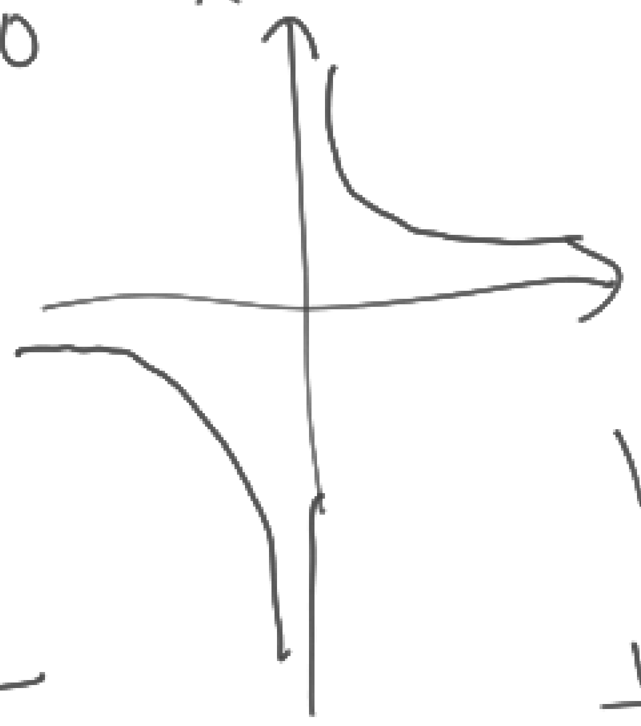
what if $p(a) = 0$ and $q(a) = 0$? We can simplify within the limit

Ex: $\lim\limits_{x\to 1} \dfrac{x^2-1}{x-1} = \lim\limits_{x\to 1}\left[\dfrac{(x+1)(x-1)}{(x-1)}\right] = \lim\limits_{x\to 1} (x+1) = 1+1 = 2$

"$= \dfrac{0}{0}$" indeterminate form (or $\infty - \infty$, $\pm\dfrac{\infty}{\infty}$, 1^∞)

Ex: $\lim\limits_{x\to 0} \dfrac{1}{x}$ "$= \dfrac{1}{0}$" DNE since $\lim\limits_{x\to 0^-} \dfrac{1}{x} = -\infty$

$\lim\limits_{x\to 0^+} \dfrac{1}{x} = +\infty$



VA at $x=0$

$\dfrac{1}{-0.001} = -1000$ $\quad -$

$\dfrac{1}{0.001} = 1000$ $\quad +$

prod = -6 6, -1

sum = 5

Ex: $\lim_{x \to 1} \frac{x^2+5x-6}{x^2+4x-5} = \frac{0}{0}$ $= \lim_{x \to 1} \frac{(x+6)(x-1)}{(x+5)(x-1)} = \lim_{x \to 1} \left(\frac{x+6}{x+5}\right) = \frac{1+6}{1+5} = \frac{7}{6}$

$\lim_{x \to -2} \frac{x^3+8}{x^2+5x+6} = \frac{0}{0}$ $= \lim_{x \to -2} \frac{(x+2)(x^2-2x+4)}{(x+2)(x+3)} = \lim_{x \to -2} \frac{x^2-2x+4}{x+3} =$

prod 6 2, 3

sum 5

$= \frac{(-2)^2-2(-2)+4}{-2+3}$

$= \frac{4+4+4}{1} = 12$

Grouping

$2x^2 - 5x - 3$

sum = -5

prod = 2(-3) = -6

-6, 1

$2x^2 - 6x + 1x - 3$

$(2x^2 - 6x) + (x - 3)$

$2x(x-3) + 1(x-3)$

$= (2x+1)(x-3)$

Ex:

a) $\lim_{x \to 1} \frac{x+5}{x^2-2x+1} = \frac{6}{0}$ so VA $= \infty$

check L, R

$\lim_{x \to 1^-} \frac{x+5}{\underbrace{x^2-2x+1}_{(x-1)^2}}$ $\frac{0.9+5}{(0.9-1)^2}$ $\frac{+}{+}$ $+\infty$

$\frac{1.1+5}{(1.1-1)^2} = \frac{+}{+} \Rightarrow \infty$

equal

b) $\lim_{x \to 3} \frac{x^3-27}{x^2-5x+6} = \frac{0}{0}$

$= \lim_{x \to 3} \left(\frac{\cancel{(x-3)}(x^2+3x+9)}{\cancel{(x-3)}(x-2)} \right) = \frac{3^2+3 \cdot 3+9}{3-2} = 27$

c) $$\lim_{h\to 0} \frac{\frac{2}{2}\left(\frac{1}{2+h}\right) - \frac{1}{2}\left(\frac{2+h}{2+h}\right)}{h} = \frac{0}{0}$$

$$= \lim_{h\to 0} \left(\frac{\frac{2}{2(2+h)} - \frac{2+h}{2(2+h)}}{h} \right)$$

$$= \lim_{h\to 0} \frac{\frac{2-2-h}{2(2+h)}}{h} = \lim_{h\to 0} \frac{-\cancel{h}}{2(2+h)\cancel{h}} = \lim_{h\to 0} \frac{-1}{2(2+h)} = -\frac{1}{4}$$