Limit Rules

$$\lim_{x\to a} f(x) = f(a) \quad \text{when } f(x) \text{ is a polyn.}$$

Suppose $\lim_{x\to a} f(x)$, $\lim_{x\to a} g(x)$ both exist.

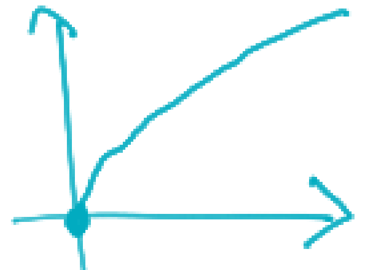
Then
$$\lim_{x\to a} f(x) \boxed{\begin{smallmatrix}+\\-\\\cdot\end{smallmatrix}} \lim_{x\to a} g(x) = \lim_{x\to a} \left(f(x) \boxed{\begin{smallmatrix}+\\-\\\cdot\end{smallmatrix}} g(x)\right)$$

$$\text{"} \div \text{"} = \text{"} \quad \text{"} \div \text{"} \quad \text{provided } \lim_{x\to a} g(x) \neq 0$$

$$\lim_{x\to a} \left(f(x)^n\right) = \left(\lim_{x\to a} f(x)\right)^n$$

Consider $\lim_{x\to 0^-} \sqrt{x}$ DNE

$\lim_{x\to 0^+} \sqrt{x} = 0$



$\sqrt{0} = 0$

$$\lim_{x\to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)} \quad \text{provided } \lim_{x\to a} f(x) > 0, \ n \text{ is even}$$

To evaluate limits: 1) Substitute in the value

GENERALLY 2) Simplify

3) Try step 1 again

eventually you won't get an indeterminate form

$\frac{0}{0}$, $\infty - \infty$, 1^∞, $0 \cdot \infty$, $\frac{\infty}{\infty}$

Ex: $\lim_{x \to 4} \frac{\sqrt{x} - 2}{x - 4} = 0/0$ (Rationalize numerator)

$= \lim_{x \to 4} \frac{(\sqrt{x} - 2)(\sqrt{x} + 2)}{(x-4)(\sqrt{x}+2)}$

Use: $(a-b)(a+b) = a^2 - b^2$

conjugates

$= \lim_{x \to 4} \frac{(\sqrt{x})^2 - 2^2}{(x-4)(\sqrt{x}+2)} = \lim_{x \to 4} \frac{\cancel{(x-4)}\, 1}{\cancel{(x-4)}(\sqrt{x}+2)}$

$= \frac{1}{\sqrt{4}+2} = \frac{1}{2+2} = \frac{1}{4}$

$$\text{Ex: } \lim_{x\to 1} \frac{\overbrace{\sqrt{3+x}}^{a} - \overbrace{\sqrt{5-x}}^{b}}{3x-3} = 0/0 \cdot \frac{\sqrt{3+x}+\sqrt{5-x}}{\sqrt{3+x}+\sqrt{5-x}}$$

$a^2 - b^2$

$$= \lim_{x\to 1} \frac{(3+x)-(5-x)}{\underbrace{(3x-3)}_{3(x-1)}(\sqrt{3+x}+\sqrt{5-x})}$$

$$(3+x)-(5-x)$$
$$= 3+x-5+x$$
$$= 2x-2$$
$$= 2(x-1)$$

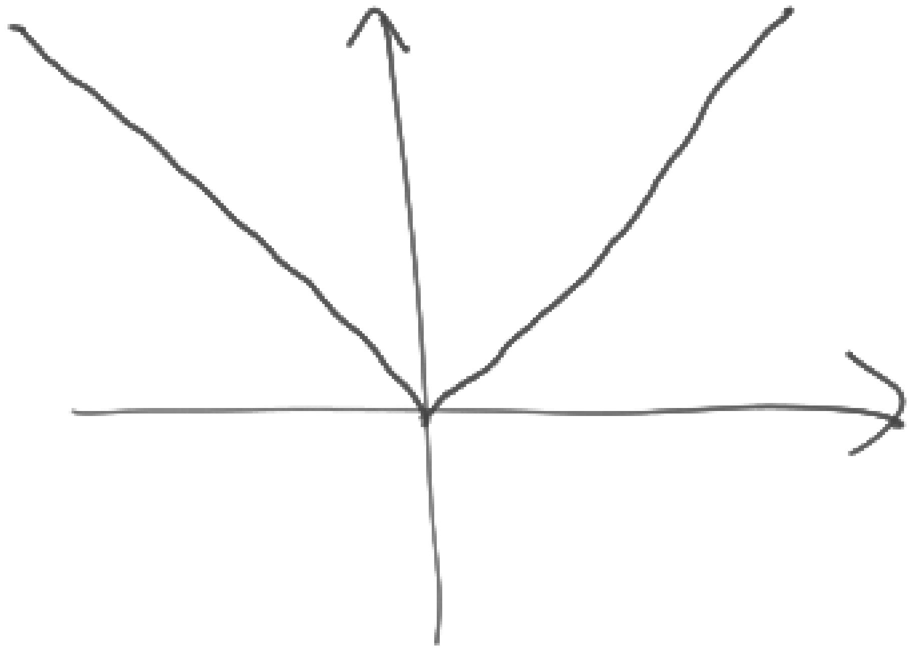
$$= \lim_{x\to 1} \frac{2\cancel{(x-1)}}{3\cancel{(x-1)}(\sqrt{3+x}+\sqrt{5-x})}$$

$$= \lim_{x\to 1} \frac{2}{3(\sqrt{3+x}+\sqrt{5-x})}$$

$$= \frac{2}{3(\sqrt{3+1}+\sqrt{5-1})} = \frac{2}{3(\sqrt{4}+\sqrt{4})}$$

$$= \frac{2}{3(2+2)} = \frac{2}{3(4)} = \frac{1}{6}$$

$$|x| = \begin{cases} -x & \text{for } x < 0 \\ x & \text{for } x \geq 0 \end{cases}$$

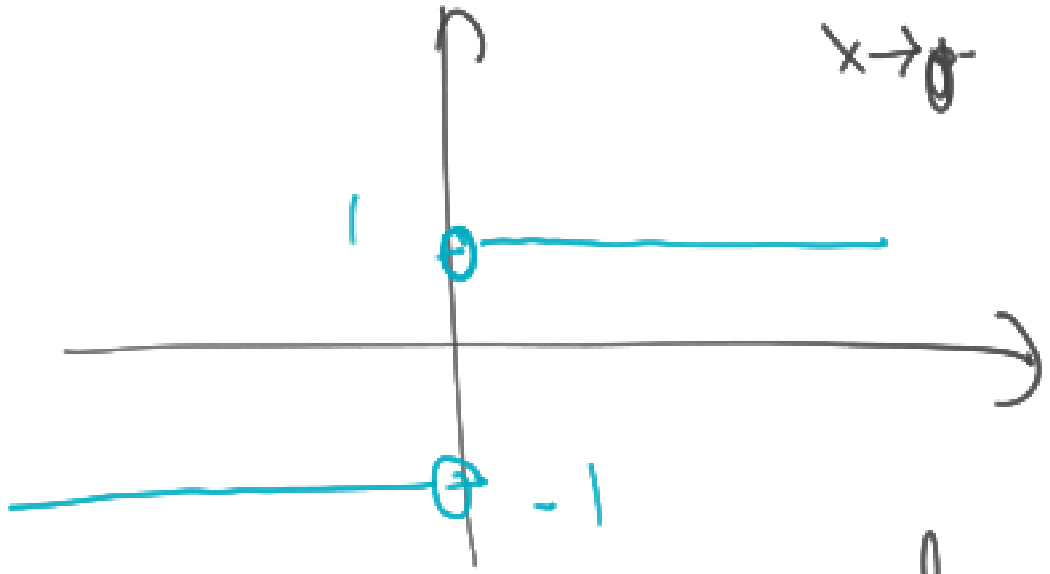


$$\frac{|x|}{x} = \begin{cases} \frac{-x}{x} = -1 & \text{for } x < 0 \\ \frac{x}{x} = 1 & \text{for } x \geq 0 \end{cases}$$

$\boxed{x \neq 0}$

Ex: $f(x) = \dfrac{|x|}{x}$

x	-3	-2	-1	0	1	2	3
y	-1	-1	-1	X	1	1	1



$$\lim_{x \to 0^-} \frac{|x|}{x} = -1$$

$$\lim_{x \to 0^+} \frac{|x|}{x} = 1$$

$$\lim_{x \to 0} \frac{|x|}{x} \text{ DNE}$$

$$|\star| = \begin{cases} -\star & \text{whn } \star < 0 \\ \star & \text{whn } \star \geq 0 \end{cases}$$

$$|2x+5| = \begin{cases} -(2x+5) & \text{whn } 2x+5<0 \rightarrow x < -5/2 \\ 2x+5 & \text{wh } 2x+5 \geq 0 \rightarrow x \geq -5/2 \end{cases}$$

Ex: $\lim_{x\to 3^-} \frac{|2x-6|}{x-3}$ $= 0/0$ problem $|:|=0$

____ either $\frac{2x-6}{x-3}$ OR $\boxed{\frac{-(2x-6)}{x-3}}$

$x \sim 2.9$ $\quad 2(2.9)-6) = - < 0$

$$= \lim_{x\to 3^-} \frac{-(2x-6)}{x-3} = \lim_{x\to 3} \frac{-2\cancel{(x-3)}}{\cancel{(x-3)}} = -2$$

$$\lim_{x\to 3^+} \frac{|2x-6|}{x-3} = 2$$

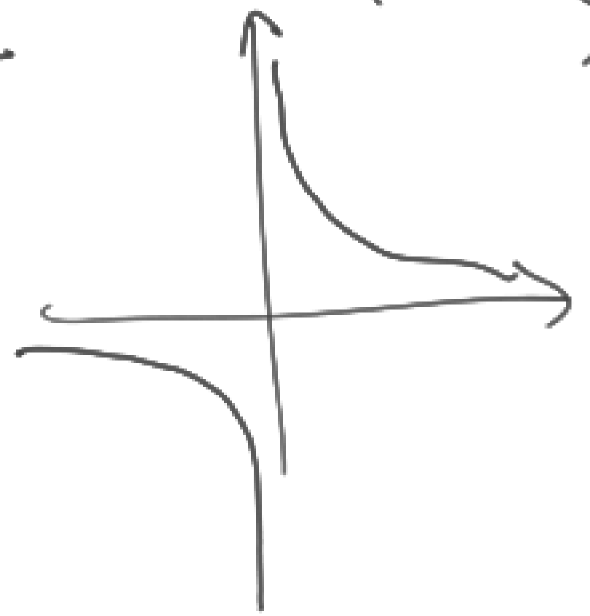
Defn: $f(x)$ has a VA at $x=a$ iff $\lim_{x\to a^-} f(x) = \pm\infty$

AND/OR $\lim_{x\to a^+} f(x) = \pm\infty$

Note: If you get $\lim f(x) = \frac{k}{0}$ then it's a VA. $(k \neq 0)$

Ex:

$f(x) = \frac{1}{x}$



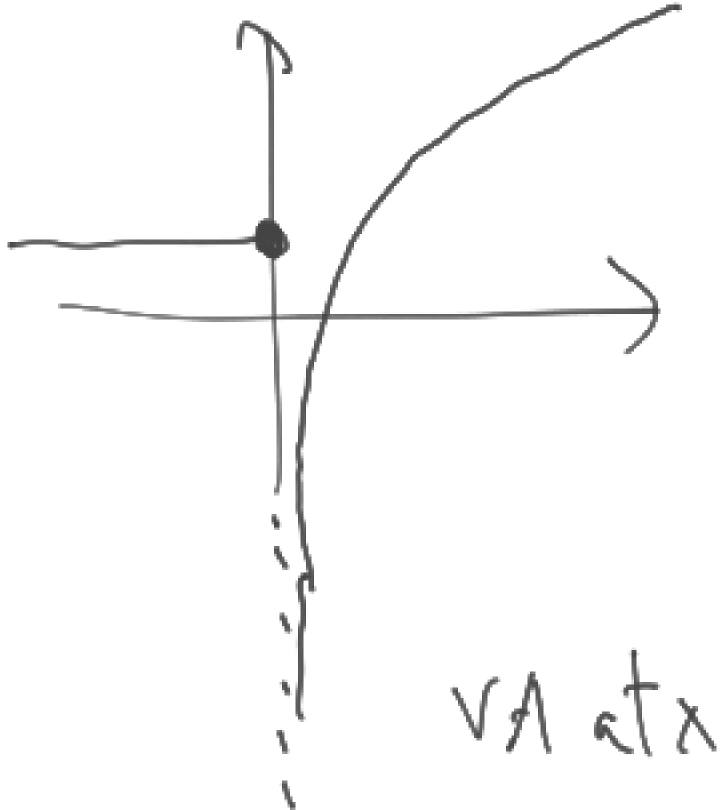
VA at $x=0$

$g(x) = \frac{1}{(x-1)^2}$

VA $x=1$

$h(x)$

$h(x) = \begin{cases} -1, & x \le 0 \\ \ln x, & x > 0 \end{cases}$



VA at $x=0$

Limits at infinity

$\lim\limits_{x \to -\infty} f(x)$ $\qquad$ $\lim\limits_{x \to \infty} f(x)$

HAs (if they exist)

horizontal asymptotes

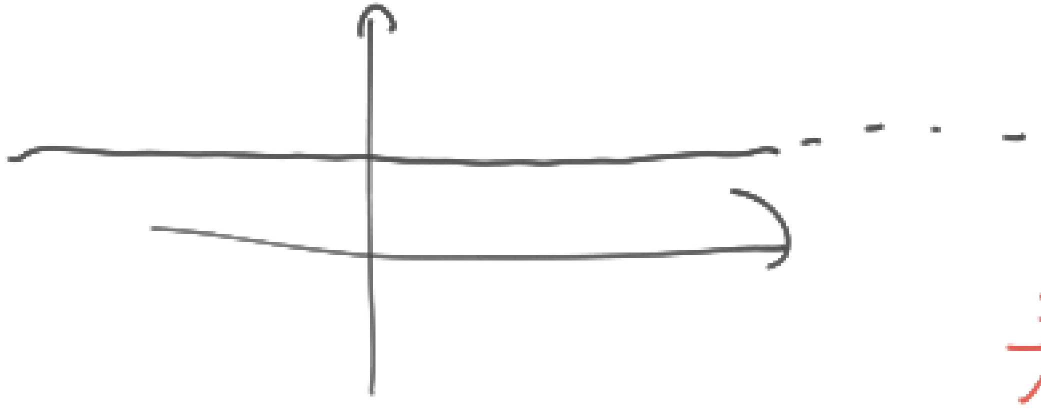
Note: $\lim\limits_{x \to -\infty} \frac{1}{x} = 0$ $= \frac{-1}{\infty}$

$\lim\limits_{x \to \infty} \frac{1}{x} = 0$ $= \frac{1}{\infty}$

$\pm \frac{k}{\infty} = 0$

Note: For rational expressions, HAs are equal (when they exist) (e.g. not $\pm\infty$)

Ex: $\lim\limits_{x \to \infty} 1 = 1$



$\lim\limits_{x \to \infty} \frac{1}{x^2} = \frac{1}{\infty^2} = 0$

$\lim\limits_{x \to \infty} \frac{x^3 - 8}{x^3 + x^2 - 5x} = \frac{\infty - 8}{\infty + \infty - \infty}$??

$\ast \rightarrow$ (SIMPLIFY BY FACTORING LARGEST POWER OF X)

$$\lim_{x \to \infty} \frac{x^3 - 8}{x^3 - x^2 - 5x} = \lim_{x \to \infty} \frac{\cancel{x^3}\left(1 - \frac{8}{x^3}\right)}{\cancel{x^3}\left(1 + \frac{1}{x} - \frac{5}{x^2}\right)} = \lim_{x \to \infty} \frac{1 - 0}{1 + 0 - 0} = 1$$

$$8/\infty = 0$$

$$\frac{1}{\infty} = 0 \qquad \frac{-5}{\infty} = 0$$