Ex: $\lim\limits_{x\to 2} \dfrac{x^5-32}{x^2-4x+4} = 0/0$

DNE

one-sided limits not equal

$p(a)=0 \quad x-a$ is a factor of $p(x)$

we know $x-2$ is a factor of x^5-32

of x^2-4x+4

$$
\begin{array}{r|l}
 & x^4+2x^3+4x^2+8x+16 \\
x-2 & x^5+0x^4+0x^3+0x^2+0x-32 \\
 & -(x^5-2x^4) \\
 & 2x^4+0x^3 - - - \quad -32 \\
 & -(2x^4-4x^3) \\
 & 4x^3 \quad -32 \\
 & -(4x^3-8x) \\
 & 8x^2 \\
 & 8x^2-16x \\
 & 16x-32 \\
 & -(16x-32) \\
 & 0
\end{array}
$$

a^3-b^3

$(a-b)(a^2+ab+b^2)$

a^5-b^5

$(a-b)(a^4+a^3b+a^2b^2+ab^3+b^4)$

$a^4-b^4=(a^2-b^2)(a^2+b^2)$

$$= \lim_{x\to 2} \frac{\cancel{(x-2)}(x^4+2x^3+4x^2+8x+16)}{\cancel{(x-2)}(x-2)}$$

$$= \lim_{x\to 2} \left(\frac{x^4+2x^3+4x^2+8x+16}{x-2}\right) \leftarrow f(x)$$

$$\frac{2^4+2(2^3)+4(2^2)+8(2)+16}{2-2} = \frac{k}{0}$$

$\boxed{VA}$

"outside $\pm\infty$"

$\lim\limits_{x\to 2^-} f(x) \quad \dfrac{+}{-} = -\infty$

$\lim\limits_{x\to 2^+} f(x) = \dfrac{+}{+} = +\infty$

LIMIT DNE

Ex: $\lim_{x\to 0^-} \frac{x - |x|}{3x}$

$-x$ or x

since inside is $-$

$\lim_{x\to 0^-} \frac{x - -x}{3x}$

$= \lim_{x\to 0^-} \frac{2x}{3x} = \frac{2}{3}$

$\lim_{x\to 0^+} \frac{x - |x|}{3x}$

$= x$ since inside is $+$

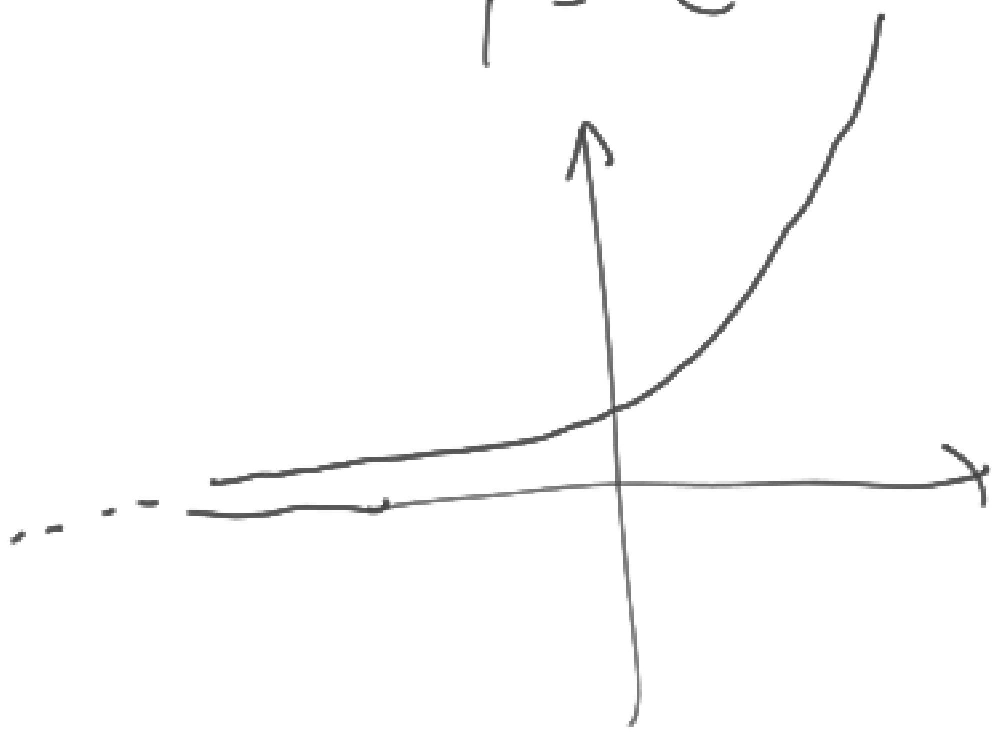
$|\bullet| = \begin{cases} -\bullet, & \bullet < 0 \\ \bullet, & \bullet \geq 0 \end{cases}$

$\lim_{x\to 0^+} \frac{x - x}{3x} = \lim_{x\to 0^+} \frac{0}{3x} = 0$

$\lim_{x\to 0} \frac{x - |x|}{3x}$ DNE

Exponential fn

$$y = e^x$$



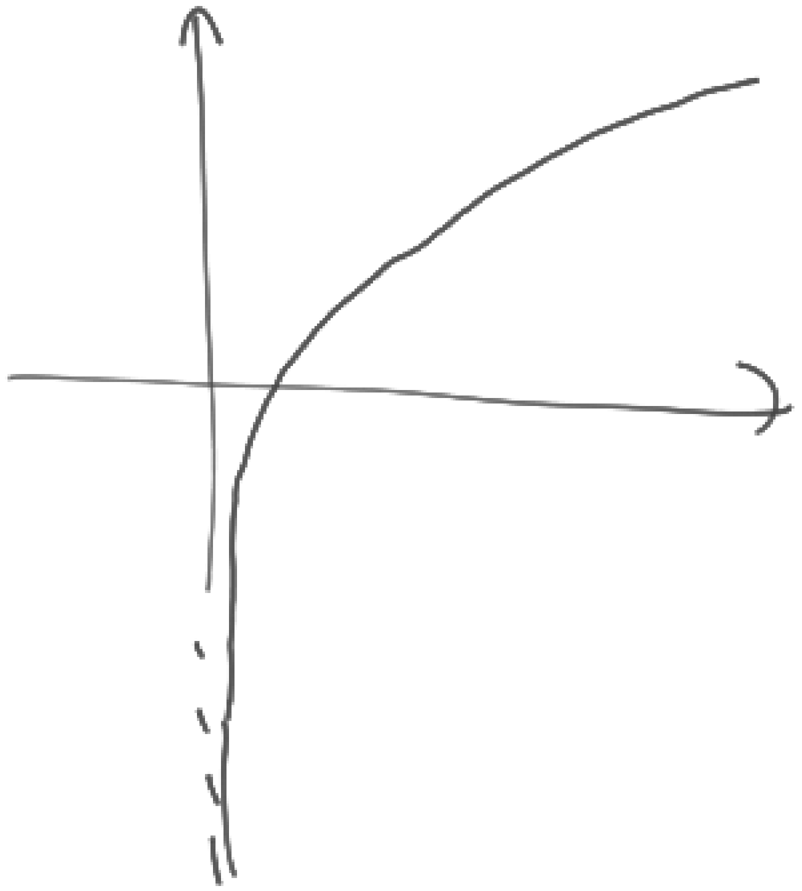
$$\lim_{x \to -\infty} e^x = 0$$

$$\lim_{x \to \infty} e^x = \infty$$

$$e^{\infty} = \infty \qquad e^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0$$

Log fn

$$y = \ln x \qquad e^{y} = x$$



$$\lim_{x \to 0^+} \ln x = -\infty$$

$$\lim_{x \to \infty} \ln x = \infty$$

Ex: $$\lim_{x\to\infty} \frac{x^5-32}{x^2-4x+4}$$

Factor out the highest growing term in the numerator and denominator

$$= \lim_{x\to\infty} \frac{x^5\left(1-32/x^5\right)}{x^2\left(1-4/x+4/x^2\right)}$$

$$= \lim_{x\to\infty} \frac{x^3\left(1-\underbrace{32/x^5}_{0}\right)}{1-\underbrace{4/x}_{0}+\underbrace{4/x^2}_{0}} = \infty^3\left(\frac{1}{1}\right) = \infty$$

$$x^4 - x$$
$$= x^4\left(1-\frac{x}{x^4}\right)$$

Ex: $$\lim_{x\to\infty} \sqrt{x^4 - x} = \lim_{x\to\infty} \sqrt{x^4\left(1-\frac{1}{x^3}\right)} = \lim_{x\to\infty} \sqrt{x^4}\sqrt{1-\frac{1}{x^3}}$$

$$= \lim_{x\to\infty} x^2\sqrt{1-\frac{1}{x^3}} = \infty^2\left(\sqrt{1}\right) = \infty$$

Ex: $\lim\limits_{x \to -\infty} \dfrac{(3x^2-5)(x+2)}{\underbrace{x(2x-1)(4x-3)}}$

$= \lim\limits_{x \to -\infty} \dfrac{3x^3+6x^2-5x-10}{8x^3-10x^2+3x}$

$= \lim\limits_{x \to -\infty} \dfrac{\cancel{x^3}\left(3+\frac{6}{x}-\frac{5}{x^2}-\frac{10}{x^3}\right)}{\cancel{x^3}\left(8-\frac{10}{x}+\frac{3}{x^2}\right)}$

$= \dfrac{3}{8}$

$\lim\limits_{x \to -\infty} \dfrac{3x^3}{8x^3}$

$= \dfrac{3}{8}$

only look at fastest growing terms

$= \lim\limits_{x \to \infty} \dfrac{3\cancel{x^3} + \underbrace{\text{smaller stuff}}}{8\cancel{x^3} + \underbrace{\text{smaller stuff}}}$

$= \dfrac{3}{8}$

Where to worry about one-sided limits

- Junctions of piecewise fns (e.g. $|0| = 0$)
- when $\sqrt{0} = 0$

  $\sqrt[2n]{0} = 0$
- $\ln|0|$

  0
- VAs