PIECEWISE FNS

Ex: $f(x) = \begin{cases} \dfrac{x^2-4}{x-2}, & x<2 \\ 3, & x=2 \\ 5-\dfrac{2}{x}, & x>2 \end{cases}$

$\lim_{x\to 2} f(x)$ SEAM

first find $\lim_{x\to 2^-} f(x)$ & $\lim_{x\to 2^+} f(x)$

$= \lim_{x\to 2^-} \dfrac{x^2-4}{x-2} = 0/0$

$= \lim_{x\to 2^-} \dfrac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}}$

$= \lim_{x\to 2^-} x+2$

$= 4$

$= \lim_{x\to 2^+} 5-\dfrac{2}{x}$

$= 5-\dfrac{2}{2} = 4$

$\lim_{x\to 2} f(x) = 4$

Defn: a fn is cts at $x=a$ if and only if $\lim_{x\to a} f(x)$, $f(a)$ both exist and are equal. ($\neq \pm\infty$)

Types of discontinuities

VA or infinite discty

JUMP $\lim_{x\to a^-} f(x) \neq \lim_{x\to a^+} f(x)$

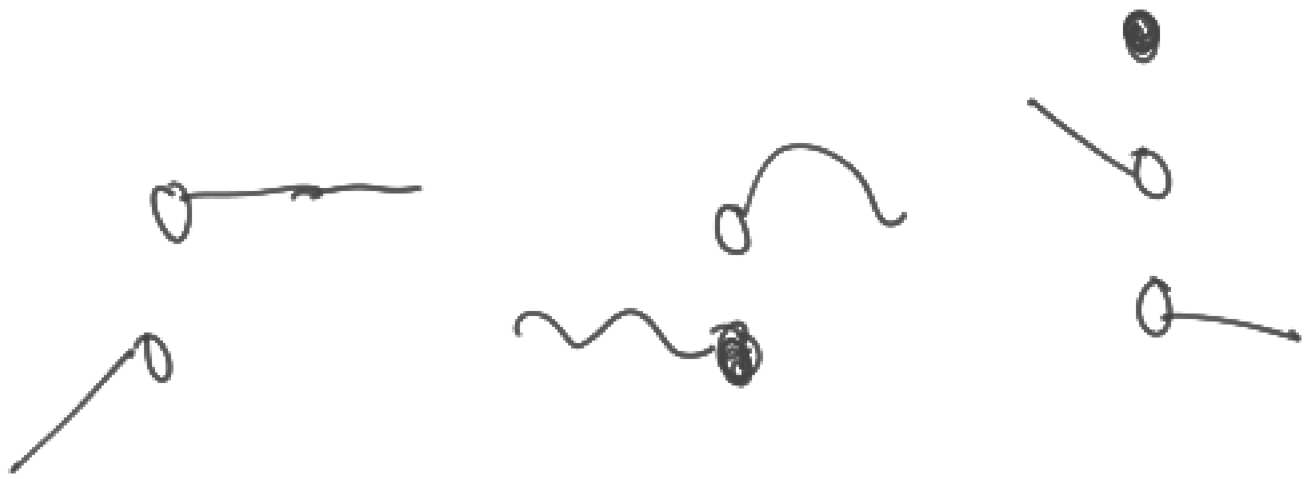
Removable $\lim_{x\to a} f(x)$ exists. $f(a)$ DNE or not equal to limit.

VA
inf

(One of the one-sided limits is $\pm\infty$)

non-removable discontinuities

JUMP



Removable

Ex: $$\lim_{x\to\infty} \frac{e^x+2}{1-2e^x} = \lim_{x\to\infty} \frac{\cancel{e^x}\left(1+\frac{2}{e^x}\right)}{\cancel{e^x}\left(\frac{1}{e^x}-2\right)} = \frac{1+\frac{2}{\cancel{\infty}}}{\frac{1}{\cancel{\infty}}-2} = -\frac{1}{2}$$

$y = e^x$

$$\lim_{x\to-\infty} \frac{e^x+2}{1-2e^x} = \frac{0+2}{1-2(0)} = 2$$

Defn: A fn is cts on $\mathbb{R}$ (or everywhere) iff $\lim_{x \to a} f(x) = f(a)$ for all $a \in \mathbb{R}$

Defn: A f is left-cts at $x = a$ iff $\lim_{x \to a^-} f(x) = f(a)$

Defn: A fn is right-cts at $x = a$ iff $\lim_{x \to a^+} f(x) = f(a)$

Defn: A fn is cts on $[a, b]$ iff f is cts for all c, $a < c < b$ and right-cts at a, left cts at b.

Ex: $\lim_{x\to\infty} \frac{x-e^x}{e^{2x}+1} = \lim_{x\to\infty} \frac{e^x\left(\frac{x}{e^x}-1\right)}{e^{2x}\left(1+\frac{1}{e^{2x}}\right)}$

$= \lim_{x\to\infty} \frac{1}{e^x}\frac{\left(\frac{x}{e^x}-1\right)}{\left(1+\frac{1}{e^{2x}}\right)}$

$= \frac{1}{\infty}\frac{(0-1)}{(1+0)} = 0$

$e^x \gg x^n$ $\boxed{\lim_{x\to\infty} \frac{x^n}{e^x} = 0}$

$e^x e^x = e^{x+x} = e^{2x}$

$x^2 x^3 = x^{2+3} = x^5$

$\lim_{x\to\infty} \frac{x-\boxed{e^x}}{\boxed{e^{2x}}+1}$

$= \lim_{x\to\infty} \frac{-e^x}{e^{2x}} = \lim_{x\to\infty} \frac{-1}{e^x} = 0$