Ex: $f(x) = \begin{cases} x+3, & x<2 \\ cx^2-1, & x \geq 2 \end{cases}$

For what values of c (if any) is f(x) cts everywhere.

Since both pieces are polyns, only need to worry about $x=2$

Need $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = f(2)$

$\lim_{x\to 2^-} x+3 = \lim_{x\to 2^+} cx^2-1 = c(2)^2-1$

$2+3 = c(2)^2-1 = c(2)^2-1$

$5 = c(2)^2-1$

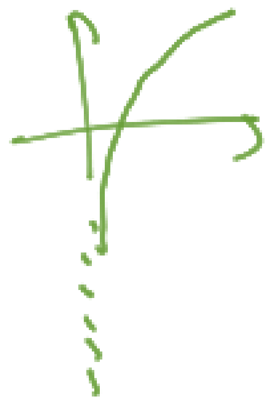
$\boxed{\text{SOLVE for c}}$

$5 = 4c-1$

$6 = 4c$

$\frac{6}{4} = c$ $\qquad c = \frac{3}{2}$

$y = \ln x$



Ex: Find all disc'ties of $g(x) = \begin{cases} \dfrac{1}{x^2-1}, & x \le 0 \\ \ln x, & 0 < x < 1 \\ \dfrac{x^2-5x+6}{x-3} & x \ge 1 \end{cases}$ and identify types

ANS

$x=-1$	INF / VA
$x=0$	INF / VA
$x=1$	JUMP
$x=3$	REM

STEPS: - check each piece
- check each seam

$\boxed{x \le 0}$

$\dfrac{1}{x^2-1} = \dfrac{1}{(x-1)(x+1)}$ VA at $x=-1$, ~~1~~ rejected

$\boxed{0 < x < 1}$ NO discs

$y = \ln x$

$\boxed{x \ge 1}$

$\dfrac{x^2-5x+6}{x-3} = \dfrac{\cancel{(x-3)}(x-2)}{\cancel{(x-3)}}$ disc at $x=3$ rem.

$x \ne 3$ $f(3)$ DNE

Check seams

$\boxed{x=0}$

$\lim_{x\to 0^-} g(x) \quad \lim_{x\to 0^+} g(x) \quad g(0)$

$= \lim_{x\to 0^-} \dfrac{1}{x^2-1} \overset{?}{=} \lim_{x\to 0^+} \ln x \overset{?}{=} \dfrac{1}{0^2-1}$ NOT all equal

$-1 \qquad -\infty \qquad -1$ disc VA inf

$\boxed{x=1}$

$\lim_{x\to 1^-} \ln x \qquad \lim_{x\to 1^+} \dfrac{x^2-5x+6}{x-3} \qquad \dfrac{1^2-5(1)+6}{1-3} = \dfrac{2}{-2}$

$0 \qquad -1 \qquad -1$ DISCTY JUMP

Ex: Consider $h(x) = \begin{cases} \frac{1}{x-3}, & x < 0 \\ x^2 - cx + 2, & x \geq 0 \end{cases}$

For what c is $h(x)$ cts everywhere.

when $x<0$, $\frac{1}{x-3}$ is cts

$x^2 - cx + 2$ is always cts.

$\lim_{x \to 0^-} h(x) = \lim_{x \to} \frac{1}{x-3} = -\frac{1}{3}$

$h(0) = \lim_{x \to} h(x) = 0^2 - 0c + 2$

$-\frac{1}{3} = 2$?

NO sol'n

Never

Ans

$$h(x) = \begin{cases} \dfrac{1}{x-3}, & x<2 \\ x^2 - cx + 2, & x \geq 2 \end{cases}$$

when is h cts? When $c = 7/2$

similarly only check the seams

$$\lim_{x\to 2^-} h(x) = \lim_{x\to 2^-} \frac{1}{x-3} = -1$$

$$\lim_{x\to 2^+} h(x) = \lim_{x\to 2^+} x^2 - cx + 2 = h(2) = 6 - 2c$$

$-1 = 6 - 2c$

$-7 = -2c$

$c = \frac{7}{2}$

$f(\square) = \sqrt{\square}$

Ex: $f(x) = \sqrt{x}$

$$\lim_{h\to 0} \frac{f(4+h) - f(4)}{h} = \lim_{h\to 0} \frac{\sqrt{4+h} - \sqrt{4}}{h} = \lim_{h\to 0} \frac{\sqrt{4+h} - 2}{h}$$

$$\lim_{h\to 0} \frac{\sqrt{4+h} - 2}{h} \cdot \frac{(\sqrt{4+h} + 2)}{(\sqrt{4+h} + 2)} = \lim \frac{(4+h) - 4}{h(\sqrt{4+h} + 2)} = \lim \frac{h}{h(\sqrt{4+h} + 2)} = \lim_{h\to 0} \frac{1}{\sqrt{4+h} + 2}$$

$$= \frac{1}{\underbrace{\sqrt{4}}_{2} + 2} = \frac{1}{4}$$

Ex: $g(x) = \frac{1}{2x+1}$

$g(\boxed{\ }) = \frac{1}{2(\boxed{\ })+1}$

$g(3) = \frac{1}{2(3)+1} = \frac{1}{7}$

$g(3+h) = \frac{1}{2(3+h)+1} = \frac{1}{7+2h}$

$g(x+h) = \frac{1}{2(x+h)+1}$

$$\lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{2(x+h)+1} - \frac{1}{2x+1}}{h} \cdot \frac{(2(x+h)+1)(2x+1)}{(2(x+h)+1)(2x+1)}$$

$$= \lim_{h \to 0} \frac{(2x+1) - (2(x+h)+1)}{h(2(x+h)+1)(2x+1)} = \lim_{h \to 0} \frac{-2\cancel{h}}{\cancel{h}(2(x+h)+1)(2x+1)}$$

$$\lim_{h \to 0} \frac{-2}{(2(x+h)+1)(2x+1)}$$

$$= \frac{-2}{(2x+1)(2x+1)}$$

$$= \frac{-2}{(2x+1)^2}$$