Practice Problems on Linear Independence **Math 201-105-RE**

1. Let $\mathbf{u_1} = \langle 6, -2, 4\rangle$, $\mathbf{u_2} = \langle -9, 3, -6\rangle$, $\mathbf{u_3} = \langle 6, 2, 4\rangle$, $\mathbf{u_4} = \langle 1, 1, 1\rangle$

For each of the following sets of vectors determine if the set if linearly dependent (LD) or independent (LI).

In each case, determine if the span is a point, line, plane or $\mathbb{R}^3$.

a. $\{\mathbf{u_1}\}$ b. $\{\mathbf{u_1}, \mathbf{u_2}\}$ c. $\{\mathbf{u_1}, \mathbf{u_2}, \mathbf{u_3}\}$ d. $\{\mathbf{u_2}, \mathbf{u_3}, \mathbf{u_4}\}$ e. $\{\mathbf{u_3}, \mathbf{u_4}\}$ f. $\{\mathbf{0}\}$.

2. Let $\mathbf{u_1}$, $\mathbf{u_2}$, $\mathbf{u_3}$, and $\mathbf{u_4}$ be any vectors in $\mathbb{R}^n$.

Determine whether the following statements are always (A), sometimes (S), or never (N) true.

a. $\mathbf{u_1} \in \operatorname{span}\{\mathbf{u_1}, \mathbf{u_2}\}$

b. $\mathbf{u_1} \in \operatorname{span}\{5\mathbf{u_1}\}$

c. If $\mathbf{u_1} \in \operatorname{span}\{\mathbf{u_2}\}$, then $\mathbf{u_2} \in \operatorname{span}\{\mathbf{u_1}\}$.

d. If $\mathbf{u_1} \in \operatorname{span}\{\mathbf{u_2}, \mathbf{u_3}\}$, then $\mathbf{u_1} \in \operatorname{span}\{\mathbf{u_2}, \mathbf{u_3}, \mathbf{u_4}\}$.

e. If $\mathbf{u_1} \in \operatorname{span}\{\mathbf{u_2}, \mathbf{u_3}, \mathbf{u_4}\}$, then $\mathbf{u_1} \in \operatorname{span}\{\mathbf{u_2}, \mathbf{u_3}\}$.

f. If $\mathbf{u_1} \in \operatorname{span}\{\mathbf{u_2}, \mathbf{u_3}\}$, then $\operatorname{span}\{\mathbf{u_1}, \mathbf{u_2}, \mathbf{u_3}\} = \operatorname{span}\{\mathbf{u_2}, \mathbf{u_3}\}$.

g. If $\operatorname{span}\{\mathbf{u_1}, \mathbf{u_2}, \mathbf{u_3}\} = \operatorname{span}\{\mathbf{u_2}, \mathbf{u_3}\}$, then $\mathbf{u_1} \in \operatorname{span}\{\mathbf{u_2}, \mathbf{u_3}\}$.

h. If $\{\mathbf{u_1}, \mathbf{u_2}\}$ is linearly dependent, then $\{\mathbf{u_1}, \mathbf{u_2}, \mathbf{u_3}\}$ is linearly dependent.

i. If $\{\mathbf{u_1}, \mathbf{u_2}, \mathbf{u_3}\}$ is linearly dependent, then $\{\mathbf{u_1}, \mathbf{u_2}\}$ is linearly dependent.

j. If $\{\mathbf{u_1}, \mathbf{u_2}, \mathbf{u_3}\}$ is linearly independent, then $\{\mathbf{u_1}, \mathbf{u_2}\}$ is linearly independent.

k. If $\{\mathbf{u_1}, \mathbf{u_2}\}$ is linearly independent, then $\{\mathbf{u_1}, \mathbf{u_2}, \mathbf{u_3}\}$ is linearly independent.

l. If $\{\mathbf{u_1}, \mathbf{u_2}, \mathbf{u_3}\}$ is linearly independent, then $\mathbf{u_1} \notin \operatorname{span}\{\mathbf{u_2}, \mathbf{u_3}\}$.

m. If $\mathbf{u_1} \notin \operatorname{span}\{\mathbf{u_2}, \mathbf{u_3}\}$, then $\{\mathbf{u_1}, \mathbf{u_2}, \mathbf{u_3}\}$ is linearly independent.

Answers

1a. LI. Line 1b. LD. Line 1c. LD. Plane 1d. LI. $\mathbb{R}^3$ 1e. LI. Plane. 1f. LD. Point.

2a. A 2b. A 2c. S (It's true for $\mathbf{u_1} \neq \mathbf{0}$.) 2d. A 2e. S 2f. A 2g. A 2h. A 2i. S

2j. A 2k. S 2l. A 2m. S (It's true when $\{\mathbf{u_2}, \mathbf{u_3}\}$ is L.I.)