**Worksheet on Subspaces of $\mathbb{R}^n$.** **Math 201-105-RE**

Determine if the following sets are subspaces. If not, determine which closure properties fail, namely (1) vector addition abbreviated by (VA) or (2) scalar multiplication abbreviated by (SM). For those that are subspaces, express them as a span of vectors.

1. $S = \{\langle x, y\rangle \in \mathbb{R}^2 | \ \langle x, y\rangle = t\langle 3, -2\rangle \text{ for some } t \in \mathbb{R}\}$

2. $S = \{\langle x, y\rangle \in \mathbb{R}^2 | \ \langle x, y\rangle = \langle 1-t, t\rangle \text{ for some } t \in \mathbb{R}\}$

3. $S = \{\langle x, y\rangle \in \mathbb{R}^2 | \ 5x + 7y = 0\}$

4. $S = \{\langle x, y\rangle \in \mathbb{R}^2 | \ x \geq 0\}$

5. $S = \{\langle x, y\rangle \in \mathbb{R}^2 | \ x^2 + y^2 \leq 1\}$

6. $S = \{\langle x, y\rangle \in \mathbb{R}^2 | \ x^2 + y^2 \leq 0\}$

7. $S = \{\langle x, y\rangle \in \mathbb{R}^2 | \ x^2 - y^2 \leq 0\}$

8. $S = \{\langle x, y, z\rangle \in \mathbb{R}^3 | \ x = -t, y = 3t, z = 0\}$

9. $S = \{\langle x, y, z\rangle \in \mathbb{R}^3 | \ x = 2 - t, y = 1 + 3t, z = 0\}$

10. $S = \{\langle x, y, z\rangle \in \mathbb{R}^3 | \ x = -t, y = 3t, z = 5\}$

11. $S = \{\langle x, y, z\rangle \in \mathbb{R}^3 | \ x + 2y - z = 0 \text{ and } 4x + 2y + 9z = 0\}$

12. $S = \{\langle x, y, z\rangle \in \mathbb{R}^3 | \ x + 2y - z = 2\}$

13. $S = \{\langle x, y, z\rangle \in \mathbb{R}^3 | \ x + 2y \geq z\}$

14. $S = \{\langle x, y, z\rangle \in \mathbb{R}^3 | \ x = 0\}$

15. $S = \{\langle x, y, z\rangle \in \mathbb{R}^3 | \ x = 2t - s, y = t + 4s, z = s\}$

16. $S = \{\langle x, y, z\rangle \in \mathbb{R}^3 | \ xy = z^2\}$

Note: Every subspace is the span of a set of of vectors, and vice versa. The two terms are equivalent. If one could show that a subset of $\mathbb{R}^b$ was the span of a set of vectors, then it must also be a subspace. This is true because linear combinations are built by adding multiples of vectors together AND the fact that doing it over and over again still gives us a linear combination. Remember that a linear combination of a linear combination is itself just a linear combination.

**Answers**

1. Yes. $S = \text{span}\{\langle 3, -2\rangle\}$ 2. No. VA and SM. 3. Yes. $S = \text{span}\{\langle 7, -5\rangle\}$ 4. No. SM 5. No. VA and SM.
6. Yes. $S = \text{span}\{\langle 0, 0\rangle\}$ 7. No. VA. 8 . Yes. $S = \text{span}\{\langle -1, 3, 0\rangle\}$ 9. No. VA and SM.
10. No. VA and SM. 11. Yes. $S = \text{span}\{\langle -20, 13, 6\rangle\}$ 12. No. VA and SM. 13. No. SM.
14. Yes. $S = \text{span}\{\langle 0, 1, 0\rangle, \langle 0, 0, 1\rangle\}$ 15. Yes. $S = \text{span}\{\langle 2, 1, 0\rangle, \langle -1, 4, 1\rangle\}$ 16. No. VA.