From the files of Norman Dobson
(edited by T. Gideon)
Calculus II – Final Exam Problems
Integration

Evaluate each integral.

1. $\displaystyle\int \frac{x}{\sqrt{x^2-1}}\,dx$
2. $\displaystyle\int x\cos ax\,dx$
3. $\displaystyle\int \frac{7x+1}{x^2-x-2}\,dx$
4. $\displaystyle\int_0^{\pi/2} \cos^3 x\,dx$
5. $\displaystyle\int \frac{x^3\,dx}{\sqrt{x+1}}$
6. $\displaystyle\int_0^{\infty} \frac{dx}{(x+1)^2}$
7. $\displaystyle\int_1^5 \frac{x^2+1}{\sqrt{x-1}}\,dx$
8. $\displaystyle\int \sin^6 x\,\cos^3 x\,dx$
9. $\displaystyle\int \sin\theta\,\ln(\cos\theta)\,d\theta$
10. $\displaystyle\int_1^2 \frac{dx}{\sqrt{x-1}}$
11. $\displaystyle\int_2^{\infty} \frac{dx}{(x-1)^2}$
12. $\displaystyle\int \sec^3 2\theta\,\tan^5 2\theta\,d\theta$
13. $\displaystyle\int_1^2 x^2\sqrt{2-x}\,dx$
14. $\displaystyle\int \frac{dx}{x(x+1)^2}$
15. $\displaystyle\int_0^{\sqrt{3}} \frac{dx}{(4-x^2)^{3/2}}$
16. $\displaystyle\int_{-2}^{2} \frac{dx}{(x+2)^2}$
17. $\displaystyle\int \frac{1}{x(x+2)}\,dx$
18. $\displaystyle\int \arctan\sqrt{x}\,dx$
19. $\displaystyle\int_1^{\infty} e^{-x}\,dx$
20. $\displaystyle\int \frac{x^3}{\sqrt{4-x^2}}\,dx$
21. $\displaystyle\int_0^1 \frac{dx}{x}$
22. $\displaystyle\int_1^5 (x+1)\sqrt{2x-1}\,dx$
23. $\displaystyle\int \sin^5 2\theta\,\cos^3 2\theta\,d\theta$
24. $\displaystyle\int_0^{\sqrt{2}} \frac{x^3}{\sqrt{4-x^2}}\,dx$
25. $\displaystyle\int x\arctan x\,dx$
26. $\displaystyle\int_0^3 \frac{dx}{(x-1)^2}$
27. $\displaystyle\int e^x\sin x\,dx$
28. $\displaystyle\int x^2\sqrt{x+1}\,dx$
29. $\displaystyle\int x\sin^3 x^2\,\cos^4 x^2\,dx$
30. $\displaystyle\int \cot^3 2x\csc^3 2x\,dx$
31. $\displaystyle\int_{-1}^{2} \frac{dx}{x^2}$
32. $\displaystyle\int_1^{\infty} \frac{dx}{(2x-1)^{3/2}}$
33. $\displaystyle\int 4\cos^3(\frac{x}{3})\,\sin^2(\frac{x}{3})\,dx$
34. $\displaystyle\int_0^{\pi/6} 4\sin^2(3x)\,dx$
35. $\displaystyle\int \sec^2 x\,\ln|\cot x|\,dx$
36. $\displaystyle\int \sec^4 x\,\tan^3 x\,dx$
37. $\displaystyle\int \frac{x^2}{(9-4x^2)^{3/2}}\,dx$
38. $\displaystyle\int_6^{\infty} \frac{dx}{x\sqrt{x^2-9}}$
39. $\displaystyle\int_0^5 \frac{dx}{x^2-4x+4}$
40. $\displaystyle\int \sin^2 x\,\cos^2 x\,dx$
41. $\displaystyle\int \cot^3 x\,dx$
42. $\displaystyle\int \sec^6 x\,\tan^3 x\,dx$
43. $\displaystyle\int_0^{\pi/4} \sin x\,\ln(\sec x)\,dx$
44. $\displaystyle\int \frac{\sqrt{9x^2-4}}{x}\,dx$
45. $\displaystyle\int_{2\sqrt{3}}^{\infty} \frac{dx}{x^2+4}$
46. $\displaystyle\int_0^1 \ln x\,dx$
47. $\displaystyle\int (\tan 2x)^2\sec 2x\,dx$
48. $\displaystyle\int \frac{dx}{x^2\sqrt{x^2-1}}$
49. $\displaystyle\int x^2(\ln x)^2\,dx$
50. $\displaystyle\int_0^{\pi/4} \frac{\sin^3 x}{\cos^4 x}\,dx$
51. $\displaystyle\int_0^{\infty} \frac{1}{\sqrt{e^x}}\,dx$
52. $\displaystyle\int_0^4 \frac{1}{(x-2)^2}\,dx$
53. $\displaystyle\int \sin^3 x\,\cos^4 x\,dx$
54. $\displaystyle\int \frac{dx}{e^x+e^{-x}}$
55. $\displaystyle\int xe^{-x}\,dx$

56. $\int \cos^2(\ln x)\frac{dx}{x}$

57. $\int e^{2x}\cos 3x\,dx$

58. $\int x^3\sqrt{x^2-4}\,dx$

59. $\int_{-\infty}^{\infty}\frac{dx}{4+x^2}$

60. $\int_0^2\frac{dx}{(x-1)^2}\,dx$

61. $\int\frac{\sec^2 3x}{\tan 3x}\,dx$

62. $\int\frac{x^3}{\sqrt{4x^2-9}}\,dx$

63. $\int_{-2}^{2}\frac{dx}{x^3}$

64. $\int_0^{\infty}xe^{-x}\,dx$

65. $\int\frac{5x^2-11x+5}{(x-1)^2(x-2)}\,dx$

66. $\int\ln^2 x\,dx$

67. $\int\frac{x^5}{\sqrt[3]{x^3+1}}\,dx$

68. $\int\sin^3 2\theta\,\cos 2\theta\,d\theta$

69. $\int_{-1}^{1}\frac{(\ln|x|)^3}{x}\,dx$

70. $\int_{\sqrt{5}}^{\sqrt{20}}\frac{dx}{(x^2-4)^{3/2}}$

71. $\int_{-\infty}^{0}xe^{2x}\,dx$

72. $\int x\tan^2 x\,dx$

73. $\int\tan^4 x\,dx$

74. $\int\sqrt{x^2+9}\,dx$

75. $\int\sqrt{x}\,\cos\sqrt{x}\,dx$

76. $\int_{-1}^{1}\frac{dx}{(2x+1)^3}$

77. $\int_e^{e^2}\ln x\sqrt{x\ln x-x}\,dx$

78. $\int_{-\infty}^{\infty}xe^{-|x|}\,dx$

79. $\int\csc^3 x\,\cot x\,dx$

80. $\int\sin^4(\frac{x}{4})\,dx$

81. $\int_0^{3/2}\frac{dx}{\sqrt{9-2x^2}}$

82. $\int\arcsin x\,dx$

83. $\int\sec^3\theta\,d\theta$

84. $\int\frac{x^3}{\sqrt{x^2-4}}\,dx$

85. $\int_0^{\infty}(x+1)e^{-x}\,dx$

86. $\int\tan^3 2x\,\sec^4 2x\,dx$

87. $\int_{1/\sqrt{2}}^{\sqrt{3}/2}\frac{\arccos x}{\sqrt{1-x^2}}\,dx$

88. $\int\frac{\tan x}{\ln\sec x}\,dx$

89. $\int e^x\sin 2x\,dx$

90. $\int_0^1\frac{t^2}{(25-9t^2)^{3/2}}\,dt$

91. $\int\frac{\arctan x}{x^2}\,dx$

92. $\int\frac{\sin x\,e^{\sec x}}{\cos^2 x}\,dx$

93. $\int_1^e x^3\ln x\,dx$

94. $\int(\arcsin x)^2\,dx$

95. $\int_0^2\frac{dx}{(x-1)^{2/3}}\,dx$

96. $\int\tan^3 2x\,\sec^3 2x\,dx$

97. $\int\frac{3x-2}{(x+1)^2(x-1)}\,dx$

98. $\int_{1/\sqrt{2}}^{\sqrt{2}}(1-t)\sqrt{2-t^2}\,dt$

99. $\int_1^{\infty}\frac{\arctan x}{x^2+1}\,dx$

100. $\int_1^{\sqrt{2}}x\,\text{arcsec}\,x\,dx$

101. $\int e^{x/2}\cos 3x\,dx$

102. $\int\frac{\sec^4 x}{\sqrt{\tan x}}\,dx$

103. $\int\frac{x^2}{x^4-1}\,dx$

104. $\int_0^1\frac{dx}{(3x-1)^{2/3}}\,dx$

105. $\int_1^{\infty}\frac{dx}{x^2}\int_0^1\frac{dy}{y^{1/3}}$

106. $\int_0^{10}xe^{5x}\,dx$

Answers:

1. $\sqrt{x^2-1}+C$
2. $\frac{\cos ax}{a^2}+\frac{x\sin ax}{a}+C$
3. $2\ln|x+1|+5\ln|x-2|+C$
4. $\frac{2}{3}$
5. $\frac{2}{7}(x+1)^{7/2}-\frac{6}{5}(x+1)^{5/2}+2(x+1)^{3/2}-2\sqrt{x+1}+C$
6. 1 (Improper)
7. $\frac{472}{15}$ (Improper)
8. $\frac{1}{7}\sin^7 x-\frac{1}{9}\sin^9 x+C$
9. $[1-\ln|\cos\theta|]\cos\theta+C$
10. 2 (Improper)
11. 1 (Improper)
12. $\frac{1}{14}\sec^7 2\theta-\frac{1}{5}\sec^5 2\theta+\frac{1}{6}\sec^3 2\theta+C$
13. $\frac{142}{105}$
14. $\ln|x|+\frac{1}{x+1}-\ln|x+1|+C$
15. $\frac{\sqrt{3}}{4}$
16. Diverges. (Improper)
17. $\frac{1}{2}\ln|x|-\frac{1}{2}\ln|x+2|+C$
18. $(x+1)\arctan\sqrt{x}-\sqrt{x}+C$
19. $\frac{1}{e}$ (Improper)
20. $\frac{1}{3}(4-x^2)^{3/2}-4\sqrt{4-x^2}+C$
21. Diverges. (Improper)
22. $\frac{186}{5}$
23. $\frac{1}{12}\sin^6 2\theta-\frac{1}{16}\sin^8 2\theta+C$
24. $\frac{2}{3}(8-5\sqrt{2})$
25. $\frac{1}{2}(x^2+1)\arctan x-\frac{1}{2}x+C$
26. Diverges. (Improper)
27. $\frac{1}{2}e^x(\sin x-\cos x)+C$
28. $\frac{2}{7}(x+1)^{7/2}-\frac{4}{5}(x+1)^{5/2}+\frac{2}{3}(x+1)^{3/2}+C$
29. $-\frac{1}{10}\cos^5 x^2+\frac{1}{14}\cos^7 x^2+C$
30. $-\frac{1}{10}\csc^5 2x+\frac{1}{6}\csc^3 2x+C$
31. Diverges. (Improper)
32. 1 (Improper)
33. $3\sin^3(\frac{x}{3})-2\sin^5(\frac{x}{3})+C$
34. $\frac{\pi}{3}$
35. $[\ln|\cot x|+1]\tan x+C$
36. $\frac{1}{4}\tan^4 x+\frac{1}{6}\tan^6 x+C$
37. $\frac{1}{4}\frac{x}{\sqrt{9-4x^2}}-\frac{1}{8}\arcsin\frac{2x}{3}+C$
38. $\frac{\pi}{18}$ (Improper)
39. Diverges. (Improper)
40. $\frac{x}{8}-\frac{1}{32}\sin 4x+C$
41. $-\frac{1}{2}\cot^2 x+\ln|\csc x|+C$
42. $\frac{1}{4}\tan^4 x+\frac{1}{3}\tan^6 x+\frac{1}{8}\tan^8 x+C$
43. $1-\frac{1}{\sqrt{2}}(1+\ln\sqrt{2})$
44. $\sqrt{9x^2-4}-2\operatorname{arcsec}\frac{3x}{2}+C$
45. $\frac{\pi}{12}$ (Improper)
46. -1 (Improper)
47. $\frac{1}{4}(\sec 2x\tan 2x-\ln|\sec 2x+\tan 2x|)+C$
48. $\frac{\sqrt{x^2-1}}{x}+C$
49. $\frac{x^3}{27}[9\ln^2 x-6\ln x+2]+C$
50. $\frac{2-\sqrt{2}}{3}$
51. 2 (Improper)
52. Diverges. (Improper)
53. $\frac{1}{7}\cos^7 x-\frac{1}{5}\cos^5 x+C$
54. $\arctan e^x+C$
55. $-e^{-x}(x+1)+C$
56. $\frac{1}{4}[2\ln x+\sin(2\ln x)]+C$
57. $\frac{2}{13}e^{2x}\cos 3x+\frac{3}{13}e^{2x}\sin 3x+C$
58. $\frac{4}{3}(x^2-4)^{3/2}+\frac{1}{5}(x^2-4)^{5/2}+C$
59. $\frac{\pi}{2}$ (Improper)
60. Diverges. (Improper)
61. $\frac{1}{3}\ln|\tan 3x|+C$
62. $\frac{9}{16}\sqrt{4x^2-9}+\frac{1}{48}(4x^2-9)^{3/2}+C$
63. Diverges. (Improper)
64. 1 (Improper)
65. $-\frac{1}{x-1}+2\ln|x-1|+3\ln|x-2|+C$
66. $x\ln^2 x-2x\ln x+2x+C$
67. $\frac{1}{2}x^3(x^3+1)^{2/3}-\frac{3}{10}(x^3+1)^{5/3}+C$
68. $\frac{1}{8}\sin^4 2\theta+C$
69. Diverges. (Improper)
70. $\frac{\sqrt{5}}{8}$
71. $-\frac{1}{4}$ (Improper)
72. $x\tan x-\frac{1}{2}x^2-\ln|\sec x|+C$
73. $x+\frac{1}{3}\tan^3 x-\tan x+C$
74. $\frac{1}{2}x\sqrt{x^2+9}+\frac{9}{2}\ln|\sqrt{x^2+9}+x|+C$
75. $2x\sin\sqrt{x}-4\sin\sqrt{x}+4\sqrt{x}\cos\sqrt{x}+C$
76. Diverges. (Improper)
77. $\frac{2e^3}{3}$
78. 0 (Improper)
79. $-\frac{1}{3}\csc^3 x+C$
80. $\frac{3x}{8}-\sin\frac{x}{2}+\frac{1}{8}\sin x+C$
81. $\frac{\pi}{4\sqrt{2}}$
82. $x\arcsin x+\sqrt{1-x^2}+C$
83. $\frac{1}{2}\sec\theta\tan\theta+\frac{1}{2}\ln|\sec\theta+\tan\theta|+C$
84. $4\sqrt{x^2-4}+\frac{1}{3}(x^2-4)^{3/2}+C$
85. 2
86. $\frac{1}{8}\tan^4 2x+\frac{1}{12}\tan^6 2x+C$
87. $\frac{5\pi^2}{288}$
88. $\ln\ln\sec x+C$

89. $\frac{1}{5}e^x(\sin 2x - 2\cos 2x) + C$

90. $\frac{1}{27}\left[\frac{3}{4} - \arcsin\frac{3}{5}\right]$

91. $-\frac{1}{x}\arctan x + \ln|x| - \frac{1}{2}\ln(x^2+1) + C$

92. $e^{\sec x} + C$

93. $\frac{1}{16}(3e^4 + 1)$

94. $x(\arcsin x)^2 + 2\sqrt{1-x^2}\arcsin x - 2x + C$

95. 6 (Improper)

96. $\frac{1}{10}\sec^5 2x - \frac{1}{6}\sec^3 2x + C$

97. $-\frac{5}{2(x+1)} - \frac{1}{4}\ln|x+1| + \frac{1}{4}\ln|x-1| + C$

98. $\frac{\pi}{3} - \frac{\sqrt{3}}{4}(1+\sqrt{2})$

99. $\frac{3\pi^2}{32}$ (Improper)

100. $\frac{1}{4}(\pi - 2)$

101. $\frac{2}{37}e^{x/2}(\cos 3x + 6\sin 3x) + C$

102. $2\sqrt{\tan x} + \frac{2}{5}(\tan x)^{5/2} + C$

103. $\frac{1}{4}\ln|x-1| - \frac{1}{4}\ln|x+1| + \frac{1}{2}\arctan x + C$

104. $1 + \sqrt[3]{2}$ (Improper)

105. $-\frac{1}{2}$ (Improper)

106. $\frac{1}{25}(49e^{50} + 1)$