Cal II: Worksheet 1 (Mostly basic substitutions)

1. $\displaystyle\int \frac{t^2-3t+1}{\sqrt{t}}\,dt$ **2.** $\displaystyle\int \frac{y+3}{y^2+6y+4}\,dy$ **3.** $\displaystyle\int \left(\sqrt{3x}-\frac{1}{\sqrt{3x}}\right)dx$ **4.** $\displaystyle\int \frac{\ln(5x)}{x}\,dx$

5. $\displaystyle\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$ **6.** $\displaystyle\int \sqrt{x}\sqrt{1+x\sqrt{x}}\,dx$ **7.** $\displaystyle\int e^{3t}e^{4t}\,dt$ **8.** $\displaystyle\int (\ln 3)\,dx$

9. $\displaystyle\int \frac{dt}{4t^2-12t+9}$ **10.** $\displaystyle\int \frac{2u^2-3u-17}{u+3}\,du$ **11.** $\displaystyle\int (e^{2x}+1)^2\,dx$ **12.** $\displaystyle\int \frac{e^{3x}}{e^{3x}+1}\,dx$

13. $\displaystyle\int \frac{1}{1+e^{-x}}\,dx$ **14.** $\displaystyle\int \frac{3x}{(5x^2+1)^2}\,dx$ **15.** $\displaystyle\int e^{\ln x}\,dx$ **16.** $\displaystyle\int \frac{(\sqrt{x}+1)^4}{\sqrt{x}}\,dx$

17. $\displaystyle\int \frac{e^{2x}}{e^{4x}+1}\,dx$ **18.** $\displaystyle\int \frac{dx}{3x\ln^2(3x)}$ **19.** $\displaystyle\int x\cos(5x^2)\,dx$ **20.** $\displaystyle\int \frac{\sin(1/x)}{x^2}\,dx$

21. $\displaystyle\int \frac{e^{\cot x}}{\sin^2 x}\,dx$ **22.** $\displaystyle\int \frac{\sec^2\theta}{3-\tan\theta}\,d\theta$ **23.** $\displaystyle\int \sqrt{\sin x}\cos x\,dx$ **24.** $\displaystyle\int \cot 5x\,dx$

25. $\displaystyle\int \frac{\arcsin x}{\sqrt{1-x^2}}\,dx$ **26.** $\displaystyle\int \frac{dx}{x(\ln^2 x+1)}$ **27.** $\displaystyle\int \frac{\sec x+\tan x}{\sec^2 x}\,dx$ **28.** $\displaystyle\int \frac{\sin^2 x}{\cos^4 x}\,dx$

29. $\displaystyle\int (\cos^4 x-\sin^4 x)\,dx$ **30.** $\displaystyle\int (\sec x+\csc x)^2\,dx$

Answers and comments

1. $\frac{2}{5}t^{5/2}-2t^{3/2}+2t^{1/2}+C$. First write the integrand as a sum of three fractions.

2. $\frac{1}{2}\ln\left|y^2+6y+4\right|+C$. Let $u=y^2+6y+4$.

3. $\dfrac{2}{\sqrt{3}}(x^{3/2}-x^{1/2})+C$. The integrand equals

$$\sqrt{3}x^{1/2}-\frac{1}{\sqrt{3}}x^{-1/2}$$

4. $\frac{1}{2}\ln^2(5x)+C$. Let $u=\ln(5x)$.

5. $2e^{\sqrt{x}}+C$. Let $u=\sqrt{x}$.

6. $\frac{4}{9}(1+x\sqrt{x})^{3/2}+C$. Let $u=1+x\sqrt{x}=1+x^{3/2}$.

7. $\frac{1}{7}e^{7t}+C$. The integrand equals e^{7t}.

8. $(\ln 3)x+C$. The integrand is a constant.

9. $-\dfrac{1}{2(2t-3)}+C$. $4t^2-12t+9=(2t-3)^2$, so $u=2t-3$.

10. $u^2-9u+10\ln|u+3|+C$. Long division.

11. $\frac{1}{4}e^{4x}+e^{2x}+x+C$. Expand the square.

12. $\frac{1}{3}\ln(e^{3x}+1)+C$. Let $u=e^{3x}+1$. Notice that absolute values are not needed in the logarithm because $e^{3x}+1>0$.

13. $\ln(e^x+1)+C$. Multiply and divide the integrand by e^x, and then let $u=e^x+1$. Absolute values are not needed in the logarithm because $e^x+1>0$.

14. $-\dfrac{3}{10(5x^2+1)}+C$. Let $u=5x^2+1$.

15. $\frac{1}{2}x^2+C$. Property of logarithms: $e^{\ln x}=x$.

16. $\frac{2}{5}(\sqrt{x}+1)^5+C$. Let $u=\sqrt{x}+1$.

17. $\frac{1}{2}\arctan(e^{2x})+C$. Let $u=e^{2x}$.

18. $-\dfrac{1}{3\ln(3x)}+C$. Let $u=\ln(3x)$.

19. $\frac{1}{10}\sin(5x^2)+C$. Let $u=5x^2$.

20. $\cos(1/x)+C$. Let $u=1/x$.

21. $-e^{\cot x}+C$. The integrand equals $e^{\cot x}\csc^2 x$, so let $u=\cot x$.

22. $-\ln|3-\tan\theta|+C$. Let $u=3-\tan\theta$.

23. $\frac{2}{3}(\sin x)^{3/2}+C$. Let $u=\sin x$.

24. $\frac{1}{5}\ln|\sin 5x|+C$. Since $\int\cot x\,dx=\ln|\sin x|+C$, we can let $u=5x$. Alternately, notice that the integrand equals $\cos 5x/\sin 5x$ and let $u=\sin 5x$.

25. $\frac{1}{2}(\arcsin x)^2+C$. Let $u=\arcsin x$.

26. $\arctan(\ln x)+C$. Let $u=\ln x$.

27. $\sin x+\frac{1}{2}\sin^2 x+C$. The integrand equals

$$\cos^2 x\left(\frac{1}{\cos x}+\frac{\sin x}{\cos x}\right)=\cos x+\sin x\cos x$$

For the term $\sin x\cos x$, either let $u=\sin x$ or use the double-angle identity

$$\sin 2x=2\sin x\cos x$$

28. $\frac{1}{3}\tan^3 x+C$. The integrand equals

$$\frac{\sin^2 x}{\cos^2 x}\cdot\frac{1}{\cos^2 x}=\tan^2 x\sec^2 x$$

so let $u=\tan x$.

29. $\frac{1}{2}\sin 2x+C=\sin x\cos x+C$. The integrand equals

$$\begin{aligned}(\cos^2 x-\sin^2 x)(\cos^2 x+\sin^2 x)&=\cos^2 x-\sin^2 x\\&=\cos 2x\end{aligned}$$

(notice the double-angle identity).

30. $\tan x+2\ln|\tan x|-\cot x+C$. After expanding the square, only the term $2\sec x\csc x$ requires some thought. Multiplying by $\sec x\cos x=1$ shows that this term equals

$$2\sec^2 x\cdot\frac{\cos x}{\sin x}=\frac{2\sec^2 x}{\tan x}$$

so we can let $u=\tan x$.