Cal II: Worksheet 2 (More substitutions)

1. $\int \sqrt{x}\sec(x\sqrt{x})\,dx$ **2.** $\int \frac{x+1}{\sqrt{x-1}}\,dx$ **3.** $\int x^2\sqrt{1-2x}\,dx$ **4.** $\int \frac{2x-1}{\sqrt{x+3}}\,dx$

5. $\int \frac{x^2-1}{\sqrt{2x-1}}\,dx$ **6.** $\int_0^4 x\sqrt{2x+1}\,dx$ **7.** $\int x^3(3x^2-4)^{2/3}\,dx$ **8.** $\int_0^{\ln 3} \frac{e^x}{\sqrt{e^x+1}}\,dx$

9. $\int \sqrt{\sqrt{x}+4}\,dx$ **10.** $\int \frac{1}{\sqrt{e^x}}\,dx$ **11.** $\int_0^7 x\sqrt[3]{x+1}\,dx$ **12.** $\int x^8\sqrt{x^3+1}\,dx$

13. $\int x^3\sqrt{x^4+5}\,dx$ **14.** $\int (s+1)^2\sqrt{s+3}\,ds$ **15.** $\int \frac{\tan(r^{1/3}+2)}{r^{2/3}}\,dr$ **16.** $\int \frac{2t}{(1-t)^{2/3}}\,dt$

17. $\int \sqrt{e^x-1}\,dx$

Answers and comments

1. $\frac{2}{3}\ln|\sec(x\sqrt{x})+\tan(x\sqrt{x})|+C$. Let $u=x\sqrt{x}=x^{3/2}$ and use the formula

$$\int \sec u\,du=\ln|\sec u+\tan u|+C$$

2. $\frac{2}{3}(x-1)^{3/2}+4(x-1)^{1/2}+C=\frac{2}{3}(x+5)\sqrt{x-1}+C$. Let $u=x-1$. Then $x=u+1$, $dx=du$ and the integral equals

$$\int \frac{u+2}{\sqrt{u}}\,du=\int(u^{1/2}+2u^{-1/2})\,du=\tfrac{2}{3}u^{3/2}+4u^{1/2}+C$$

Alternately, let $u^2=x-1$ (i.e., $u=\sqrt{x-1}$). Then $x=u^2+1$, $dx=2u\,du$ and the integral equals

$$\int \frac{u^2+2}{u}\cdot 2u\,du=\tfrac{2}{3}u^3+4u+C$$

3. $-\frac{1}{12}(1-2x)^{3/2}+\frac{1}{10}(1-2x)^{5/2}-\frac{1}{28}(1-2x)^{7/2}+C$. Let $u=1-2x$. Then $x=\frac{1}{2}(1-u)$, $dx=-\frac{1}{2}\,du$ and the integral equals

$$\int \tfrac{1}{4}(1-u)^2\sqrt{u}\cdot(-\tfrac{1}{2})\,du=-\tfrac{1}{8}\int(u^{1/2}-2u^{3/2}+u^{5/2})\,du$$

Alternately, let $u^2=1-2x$ (i.e., $u=\sqrt{1-2x}$). Then $x=\frac{1}{2}(1-u^2)$, $dx=-u\,du$ and the integral equals

$$\int -\tfrac{1}{4}(1-u^2)^2u^2\,du=-\tfrac{1}{4}\int(u^2-2u^4+u^6)\,du$$

4. $\frac{4}{3}(x+3)^{3/2}-14(x+3)^{1/2}+C=\frac{2}{3}(2x-15)\sqrt{x+3}+C$. Let $u=x+3$. Then $x=u-3$, $dx=du$ and the integral equals

$$\int \frac{2u-7}{\sqrt{u}}\,du=\int(2u^{1/2}-7u^{-1/2})\,du=\tfrac{4}{3}u^{3/2}-14u^{1/2}+C$$

Alternately, let $u^2=x+3$ (i.e., $u=\sqrt{x+3}$). Then $x=u^2-3$, $dx=2u\,du$ and the integral equals

$$\int \frac{2u^2-7}{u}\cdot 2u\,du=\tfrac{4}{3}u^3-14u+C$$

5. $\frac{1}{20}(2x-1)^{5/2}+\frac{1}{6}(2x-1)^{3/2}-\frac{3}{4}(2x-1)^{1/2}+C$.

Let $u^2=2x-1$ (i.e., $u=\sqrt{2x-1}$). Then $x=\frac{1}{2}(u^2+1)$, $dx=u\,du$, and the integral equals

$$\int \frac{\frac{1}{4}(u^2+1)^2-1}{u}\cdot u\,du=\tfrac{1}{4}\int(u^4+2u^2-3)\,du$$

Alternately, we can let $u=2x-1$, so $x=\frac{1}{2}(u+1)$, $dx=\frac{1}{2}\,du$ and the integral equals

$$\int \frac{\frac{1}{4}(u+1)^2-1}{\sqrt{u}}\cdot\tfrac{1}{2}\,du=\tfrac{1}{8}\int(u^{3/2}+2u^{1/2}-3u^{-1/2})\,du$$

6. 298/15.
Let $u^2=2x+1$ (i.e., $u=\sqrt{2x+1}$). Then

$$x=0\implies u=1\qquad x=4\implies u=3$$
$$x=\tfrac{1}{2}(u^2-1)\implies dx=u\,du$$

and the integral equals

$$\int_1^3 \tfrac{1}{2}(u^2-1)u\cdot u\,du=\tfrac{1}{2}\int_1^3(u^4-u^2)\,du$$

Alternately, we can let $u=2x+1$, so $x=\frac{1}{2}(u-1)$, $dx=\frac{1}{2}\,du$ and the new limits of integration are $u=1$ and $u=9$.

7. $\frac{1}{48}(3x^2-4)^{8/3}+\frac{2}{15}(3x^2-4)^{5/3}+C$.
Let $u=3x^2-4$. Then

$$x^2=\tfrac{1}{3}(u+4)\implies x\,dx=\tfrac{1}{6}\,du$$

and the integral equals

$$\int x^2(3x^2-4)^{2/3}\cdot x\,dx=\int\tfrac{1}{3}(u+4)u^{2/3}\cdot\tfrac{1}{6}\,du$$

8. $4-2\sqrt{2}$. Let $u=e^x+1$. Then the integral equals

$$\int_2^4 u^{-1/2}\,du$$

9. $\frac{4}{5}(\sqrt{x}+4)^{5/2}-\frac{16}{3}(\sqrt{x}+4)^{3/2}+C$.
Let $u=\sqrt{x}+4$. Then $\sqrt{x}=u-4$, so

$$x=(u-4)^2\implies dx=2(u-4)\,du$$

The integral then equals

$$\int \sqrt{u} \cdot 2(u-4)\,du = 2\int (u^{3/2} - 4u^{1/2})\,du$$

Alternately, one can let $u^2 = \sqrt{x} + 4$ (i.e., $u = \sqrt{\sqrt{x}+4}$). Then

$$dx = 4u(u^2 - 4)\,du$$

and the integral equals

$$\int 4u^2(u^2 - 4)\,du$$

10. $-2e^{-x/2} + C$. The integrand equals $e^{-x/2}$, so let $u = -\frac{1}{2}x$.

11. 1209/28.
Let $u^3 = x + 1$ (i.e., $u = \sqrt[3]{x+1}$). Then

$$x = 0 \implies u = 1 \qquad x = 7 \implies u = 2$$
$$x = u^3 - 1 \implies dx = 3u^2\,du$$

and the integral equals

$$\int_1^2 (u^3 - 1)u \cdot 3u^2\,du = 3\int_1^2 (u^6 - u^3)\,du = 3(\tfrac{1}{7}u^7 - \tfrac{1}{4}u^4)\Big]_1^2$$

Alternately, you can let $u = x + 1$, so $x = u - 1$, $dx = du$, and the new limits of integration are 1 and 8.

12. $\frac{2}{21}(x^3+1)^{7/2} - \frac{4}{15}(x^3+1)^{5/2} + \frac{2}{9}(x^3+1)^{3/2} + C$.
Let $u = x^3 + 1$. Then $x^3 = u - 1$, $3x^2\,dx = du$ and the integral equals

$$\int (x^3)^2\sqrt{x^3+1} \cdot x^2\,dx = \int (u-1)^2\sqrt{u} \cdot \tfrac{1}{3}\,du$$

13. $\frac{1}{6}(x^4+5)^{3/2} + C$. Let $u = x^4 + 5$.

14. $\frac{2}{7}(s+3)^{7/2} - \frac{8}{5}(s+3)^{5/2} + \frac{8}{3}(s+3)^{3/2} + C$.
Let $u^2 = s + 3$ (i.e., $u = \sqrt{s+3}$). Then $s = u^2 - 3$, $ds = 2u\,du$ and the integral equals

$$\int u(u^2-2)^2 \cdot 2u\,du = 2\int (u^6 - 4u^4 + 4u^2)\,du$$

Alternately, one can let $u = s + 3$, so $s = u - 3$, $ds = du$ and the integral equals

$$\int u^{1/2}(u-2)^2\,du = \int (u^{5/2} - 4u^{3/2} + 4u^{1/2})\,du$$

15. $3\ln|\sec(r^{1/3}+2)| + C$. Let $u = r^{1/3} + 2$ and use the formula

$$\int \tan u\,du = \ln|\sec u| + C$$

16. $\frac{3}{2}(1-t)^{4/3} - 6(1-t)^{1/3} + C$.
Let $u = 1 - t$, so $t = 1 - u$, $dt = -du$ and the integral equals

$$\int -\frac{2(1-u)}{u^{2/3}}\,du = 2\int (u^{1/3} - u^{-2/3})\,du$$

17. $2\sqrt{e^x - 1} - 2\arctan\sqrt{e^x-1} + C$.
Let $u^2 = e^x - 1$ (i.e., $u = \sqrt{e^x - 1}$). Then

$$2u\,du = e^x\,dx = (u^2+1)\,dx \implies dx = \frac{2u}{u^2+1}\,du$$

and the integral equals

$$\int \frac{2u^2}{u^2+1}\,du = 2\int \left(1 - \frac{1}{u^2+1}\right)du$$

(long division).