Cal II: Worksheet 3 (mostly inverse trig integrals and completing the square)

1. $\displaystyle\int \frac{dx}{\sqrt{9-4x^2}}$

2. $\displaystyle\int \frac{x+1}{\sqrt{x^2+2x-5}}\,dx$

3. $\displaystyle\int \frac{dx}{(x+2)\sqrt{x^2+4x-5}}$

4. $\displaystyle\int \frac{dx}{\sqrt{3-2x-x^2}}$

5. $\displaystyle\int_2^5 \frac{dx}{x^2-4x+13}$

6. $\displaystyle\int_{1/\sqrt{2}}^1 \frac{1}{x\sqrt{4x^2-1}}\,dx$

7. $\displaystyle\int \frac{dx}{x(9+4\ln^2 x)}$

8. $\displaystyle\int \frac{e^x}{\sqrt{4-e^x}}\,dx$

9. $\displaystyle\int \frac{e^x}{\sqrt{4-e^{2x}}}\,dx$

10. $\displaystyle\int \frac{dx}{\sqrt{e^{2x}-4}}$

Answers and comments

For all but two of the above, we can use one of the following three integral formulas. The comments indicate which substitution u and constant a are required.

A. $\displaystyle\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\left(\frac{u}{a}\right)+C$ **B.** $\displaystyle\int \frac{du}{u^2+a^2} = \frac{1}{a}\arctan\left(\frac{u}{a}\right)+C$ **C.** $\displaystyle\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\left(\frac{u}{a}\right)+C$

1. $\frac{1}{2}\arcsin(\frac{2}{3}x)+C$. Since $9-4x^2 = 3^2-(2x)^2$, use formula A with $u=2x$ and $a=3$.

2. $\sqrt{x^2+2x-5}+C$. Let $u=x^2+2x-5$.

3. $\frac{1}{3}\operatorname{arcsec}\big(\frac{1}{3}(x+2)\big)+C$. Since

$$x^2+4x-5 = (x+2)^2-3^3,$$

use formula C with $u=x+2$ and $a=3$.

4. $\arcsin\big(\frac{1}{2}(x+1)\big)+C$. Since

$$3-2x-x^2 = 2^2-(x+1)^2,$$

use formula A with $u=x+1$ and $a=2$.

5. $\frac{1}{12}\pi$. Since $x^2-4x+13 = (x-2)^2+3^2$, use formula B with $u=x-2$ and $a=3$. With respect to u, the limits of integration are 0 and 3.

6. $\frac{1}{12}\pi$. Since $4x^2-1 = (2x)^2-1$, use formula C with $u=2x$ and $a=1$. With respect to u, the limits of integration are $\sqrt{2}$ and 2.

7. $\frac{1}{6}\arctan(\frac{2}{3}\ln x)+C$. Use formula B with $u=2\ln x$ and $a=3$.

8. $-2\sqrt{4-e^x}+C$. Let $u=e^x$.

9. $\arcsin(\frac{1}{2}e^x)+C$. Use formula A with $u=e^x$ and $a=2$.

10. $\frac{1}{2}\operatorname{arcsec}(\frac{1}{2}e^x)+C$. Use formula C with $u=e^x$ and $a=2$. Note that

$$du = e^x\,dx \implies dx = \frac{du}{e^x} = \frac{du}{u}$$