

Mathematical Puzzles, Games and Other Diversions

Day 11

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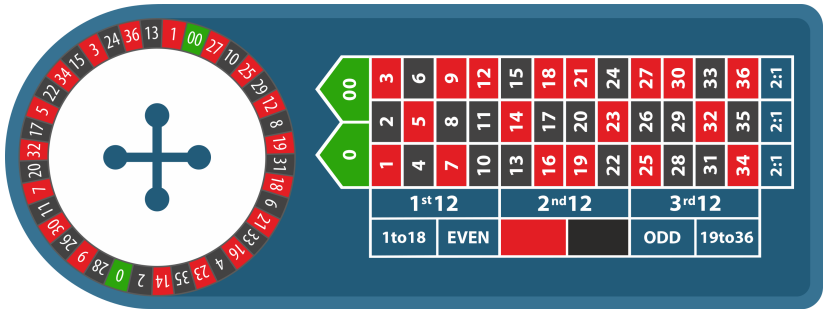
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Games of Chance

- ▶ Tic-tac-toe, chess and go are considered games of skill.
- ▶ Poker sits in a grey area (part chance, some skill).
- ▶ However, some games are clearly 100% chance.



Roulette



AMERICAN ROULETTE

- ▶ 38 Numbers: 18 BLACK, 18 RED, 2 GREEN (0 and 00)
- ▶ Every spin of the wheel is independent.
- ▶ The simplest best is RED or BLACK. It pays 1 to 1.
- ▶ What are your odds of winning the bet?
- ▶ How do we measure your disadvantage (or advantage)?

Expected Value

- ▶ It's a weighted avg of how much you can win or lose on a bet.

$$E[X] = \sum_{i=1}^k x_i p_i = x_1 p_1 + x_2 p_2 + \cdots + x_k p_k, \text{ where } p_1 + \cdots + p_k = 1$$

- ▶ We can use that formula to evaluate the “worth” of any bet (or series of bets).
- ▶ So if you bet on RED, you have 18/38 chance of winning \$1 and 20/38 chance of losing \$1 (or winning $-\$1$).

$$E[X] = 1 \left(\frac{18}{38} \right) + (-1) \left(\frac{20}{38} \right) = -\frac{1}{19} \approx -0.0526$$

- ▶ When expectation is negative, the bet is called unfavorable.

Expected Value (cont.)

- ▶ Let's say you bet on RED twice in a row.
- ▶ What is the probability that you WIN both bets?

$$\left(\frac{18}{38}\right)^2 = \frac{81}{361}$$

- ▶ What is the probability that you LOSE both bets?

$$\left(\frac{20}{38}\right)^2 = \frac{100}{361}$$

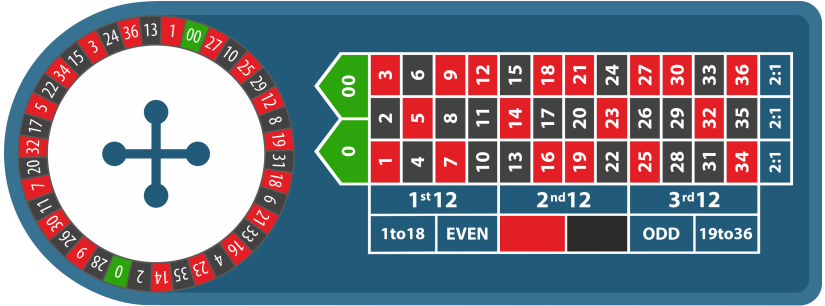
- ▶ What is the probability that you win exactly one bet?

$$\left(\frac{18}{38}\right)\left(\frac{20}{38}\right) + \left(\frac{20}{38}\right)\left(\frac{18}{38}\right) \quad \text{OR} \quad 1 - \frac{81}{361} - \frac{100}{361}$$

$$= 180/361 \approx 0.4986.$$

- ▶ What is the expected value of this strategy?

Expected Value (cont.)



AMERICAN ROULETTE

Some Other Bets

- ▶ Single Number: pays 35 to 1
- ▶ Dozen Numbers: pays 2 to 1
- ▶ Two Numbers: pays 17 to 1