Mathematical Puzzles, Games and Other Diversions

Day 14

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The Missing Dollar Riddle

- Three guests check into a hotel room.
- They're told the bill is \$30 so they pay 10 bucks each.
- The manager realizes they were overcharged, and gives the bellhop \$5 to return to the guests.
- As the bellhop goes to the room, the guests just take \$1 each, insisting on leaving a \$2 tip to the bellhop.

RECAP

- Each guest paid \$9 (after getting a dollar back).
- The bellhop got \$2. Added to the \$27, the total is \$29.
- So since the guests originally handed in \$30, what happened to the missing dollar?

Penney's Game

- Player 1 chooses a sequence of heads and tails (length ≥ 3).
- Player 2 then does the same.
- A fair coin is tossed until either player's sequence comes up. Whoever's sequence comes up first wins.

Seem fair, but it is definitely not.
Depending on P1's choices, P2 may have as much as a 7 to 1 edge.

Strategy

Once P1 chooses sequence ABC, P2 chooses (NOT B)AB.

Let's see an example of why this works.

Penney's Game (cont.)

If P1 chooses H**H**H, then P2 chooses **T**HH
HHH can only win if it occurs immediately.

That happens with probability $(1/2) \times (1/2) \times (1/2) = 1/8$.
So P2 wins 7 times out of 8.

P1 choice	P2 choice	Odds in favour of P2
H**H**H	**T**HH	7 to 1
H**H**T	**T**HH	3 to 1
H**T**H	**H**HT	2 to 1
H**T**T	**H**HT	2 to 1
T**H**H	**T**TH	2 to 1
T**H**T	**T**TH	2 to 1
T**T**H	**H**TT	3 to 1
T**T**T	**H**TT	7 to 1

Table: Analysis of length-3 game

Penney's Game (cont.)

Humble-Nishayama game

- If we apply this game to a complete deck of cards, we can pronounce the winner as the player whose sequence comes up most often.
- Player 2 should win over 80% of the time.
- Player 1 wins less than 12% of the time.

Further Discussion

- What if we look at sequences of longer length?
- Can a length-4 sequence ever beat a length-3 sequence?
- If the coin isn't fair, what can we say about the game?

The Monty Hall Problem

On a game show, and you're given the choice of three doors:
Behind one door is a car; behind the others, goats.

- You pick a door (say door #1)
- The host opens another door to show a goat (say door #3).
- He then asks, "Do you want to switch to door #2?"

Should you switch?

Standard Assumptions

- The host must always open a door that was not picked by the contestant and reveal a goat (never the car).
- The host must always offer the chance to switch between the originally chosen door and the remaining closed door.