Mathematical Puzzles, Games and Other Diversions

Day 15

Derrick Chung

March 31, 2020

Notes

Exponential Growth

Definition

We say a quantity grows exponentially when its rate of change (with respect to time) is proportional to the quantity itself.

Question:

A pond starts out with one lily pad. These pads reproduces once a day, so the area the pads cover doubles every day. The lily pads completely cover the pond after sixty days.

After how many days did they cover 1% of the area of the pond? [See https://youtu.be/fgBla7RepXU at 2:26]

Answer: After 54 days (actually ≈ 53.356)

- Let's denote by N_0 the total area covered on day 0.
- What we do know is that if we double that amount over and over again (60 times in fact), we'll cover the whole pond.

Notes

Exponential Growth (cont.)

$$\underbrace{N_0}_{\text{Area on day 0}} \quad \underbrace{\times 2 \times 2 \times \cdots \times 2}_{\text{60 times}} = N_0 \cdot 2^{60} = 100\%$$

So if we backtrack to day 54 by dividing by 2 six times, we get $100\% \div 2^6 = 1.5625\%$ of the pond covered.

Let's get an even more precise answer, and generalize the result.

- We're trying to solve for x in the equation $N_0 \cdot 2^x = 1\%$ given that we know $N_0 \cdot 2^{60} = 100\%$.
- Let's divide the first equation by the second.

  $$\frac{N_0 \cdot 2^x}{N_0 \cdot 2^{60}} = \frac{1\%}{100\%} \Rightarrow 2^{x-60} = 0.01 \Rightarrow x - 60 = \log_2(0.01)$$

  $$\Rightarrow x = 60 + \log_2(0.01)$$
- If we want to know when we reach any percentage p, the equation is $x = 60 + \log_2(p/100)$.

Notes

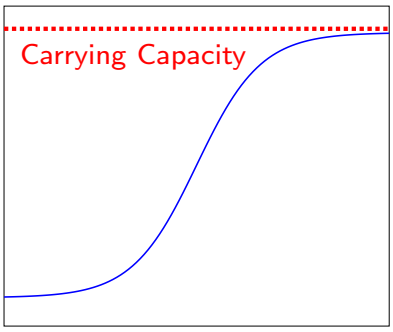
Exponential growth and epidemics

How does this relate to epidemics?

Contagious diseases generally spread exponentially (sort of).
(More details here: https://youtu.be/gxAaO2rsdIs)

Some issues to consider:

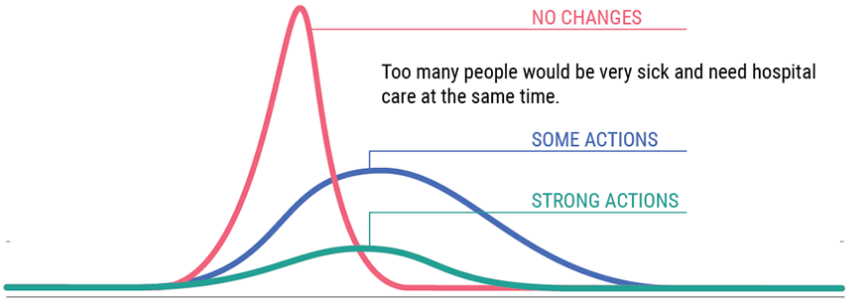
- A disease can't spread infinitely.
- The rate of infection isn't constant over time.
- Logistic growth is a better model.



Notes

Exponential growth and epidemics (cont.)

What about COVID-19? What does it mean to flatten the curve?



Don't blindly trust the analyses OR the graphs.
Keep in mind:

- Varied sources of data
- Availability and Reliability of Testing
- There's a lot we don't know about the virus and the disease.
- It's complicated.
- Everything is just a guess.

Notes

Conclusion

What now?

- Try not to overload yourself with information.
- Follow the government's guidelines.
- Adapt. This is NOT easy.
- Socialize ... from a safe distance.
- It's normal to be angry, scared, depressed.
- It's ok to ask for help, guidance.
- Be kind to others AND yourself.

Notes