Math 201-DAC-AB **Homework 1** **Due Wed Sept 14**

Fall 2022

Answers

1. Consider a similar game to 21 that we'll call 43. In this game, the players each take turns naming and adding numbers from 1 to 4, where the winner is the player who reaches 43. What's the winning strategy? And for which player? What if we want to reach the number 44? or 45? Does the answer change? Why or why not?

Answer: In 21, the important number was 4. In this case it's 5. This means whoever gets to $43 - 5 = 38$ first wins. Continuing in that logic, we just keep subtracting 5 until we can't anymore and reach our smallest target of 3. So Player 1 (P1) wins by simply playing 4 and then making sure to match P2's play with a number that brings the total of their two turns to 5.

In 44, P1 just starts with 4 to win and follows the same strategy. In 45, no matter what move P1 starts with, P2 can reach 5 first, so by applying the same basic strategy P2 will win.

2. These questions all deal with tic-tac-toe (regular or inverse). For the sake of simplicity, we assume X goes first.

(a) In inverse tic-tac-toe, given the position below, what are the best move(s) for player 2 (if any)? Why?

X	O	X

Answer: O simply needs to play on one of the three remaining edges (top middle, middle left, or middle right) to win. From here, O can guarantee a win.

Here's one simple strategy that works: No matter what X does, O just plays another edge for his next move (at least one will be open). Then if X hasn't lost yet, for O's last move, he just needs to avoid the centre, and X will lose on the last turn.

For any other starting move, X can force a tie under perfect play. For instance, if O plays the dead centre, X simply plays above him. And X's next move just needs to be in the same column as O .

If, O plays on a corner, X should play beneath him. O must avoid the top middle square or he'll lose. So X can grab it, forcing a draw.

(b) In inverse tic-tac-toe, given the position below, what are the best moves for player 1 (if any)? Why?

X	O	
	X	
		O

Answer: X must go to the centre bottom. Any other move will allow O to win. Logically, this makes sense because it's the only move that doesn't create a spot where X could complete a line.

(c) How many distinct configurations exist where X wins regular tic-tac-toe on her third move? How many distinct **games** exist where X wins on her third move (Note: these answers are different)?

Answer: There are exactly 8 ways for X to win: by completing a rows (3 ways), a column (3 ways), or a diagonal (2 ways).

X	X	X

ROW

X		
X		
X		

COLUMN

X		
	X	
		X

DIAGONAL

Since X wins in 3 moves, there will be two O's on the board.

So with 6 possible spots, there are $\binom{6}{2} = 15$ possible places for the two O's to go.

Since this is true no matter how X wins, the total number of configurations is $8 \cdot 15 = 120$.

As for distinct games, X has $3! = 6$ different orders in which he can play his three X's, and O has 2 ways to play his O's. So that means $6 \cdot 2 = 12$ total different possible game orders for each configuration. Multiplying 12 by 120 gives us 1440 total possible games ending at move 3 for X.

(d) How many different configurations exist where O wins regular tic-tac-toe on her third move with her three O's on the first row? How many different games?

O	O	O

Answer: There are six spots where X could place three X's. So $\binom{6}{3} = 20$ different configurations in all since O's choices are fixed. However, two of theses configurations are impossible. In particular, X could not have covered all of row 2 or all of row 3. Otherwise, X would have already won. So we subtract 2 from the total. This gives a total of 18.

As for total games? here they both moved three times, so each had 3! ways they could have ordered their moves, so we multiply (18 board configurations) × (6 orderings for X) × (6 orderings for O). This gives a total of 648 distinct 3-move games that end as prescribed.

3. Alice, Bob, Cheryl, Daniel and Elliott need to line up for a photograph. However, Alice and Bob hate each other and refuse to stand next to each other. How many different ways can we arrange the photograph without upsetting Alice and Bob?

Answer: It's quicker to instead consider the exact opposite situation, namely all the arrangements where Alice and Bob are next to each other. Then we can see them as one unit AB. And we just have to look at the permutations of AB,C,D and E.

There are of $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$ such combinations.

However, if Alice and Bob are next to each other, Alice might be to the left or she may be to the right. So we have to double that number, giving us $2 \cdot 24 = 48$ possible arrangements with Alice next to Bob.

Now we subtract that from the total number of permutations of 5 people, which is $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$.

This leave us $120 - 48 = 72$ possible arrangements that keep Alice and Bob apart.