Math 201-DAC-AB **Homework 2** **Due Sept 30th (by 11:30am)**

Winter 2022

Answers

1. In poker, there are 10 possible types of 5-card poker hands, as shown in class. For each type, give a formula that gives the total number of hands with that rank. Furthermore (and this is the important part), explain why that formula gives the right number.

Answer:

HAND RANKINGS	CONFIGURATION	# OF POSSIBLE HANDS
ROYAL FLUSH	A-K-Q-J-10 of same suit	4
STRAIGHT FLUSH	Five sequential cards of same suit	$10\binom{4}{1} - 4$
FOUR OF A KIND	$xxxxa$	$\binom{13}{1}\binom{4}{4} \times \binom{12}{1}\binom{4}{1}$
FULL HOUSE	$xxxyy$	$\binom{13}{1}\binom{4}{3} \times \binom{12}{1}\binom{4}{2}$
FLUSH	Five cards of same suit	$\binom{13}{5}\binom{4}{1} - \binom{10}{1}\binom{4}{1}$
STRAIGHT	Five sequential cards	$\binom{10}{1}\binom{4}{1}^5 - \binom{10}{1}\binom{4}{1}$
THREE OF A KIND	$xxxab$	$\binom{13}{1}\binom{4}{3} \times \binom{12}{2}\binom{4}{1}\binom{4}{1}$
TWO PAIRS	$xxyya$	$\binom{13}{2}\binom{4}{2}\binom{4}{2} \times \binom{11}{1}\binom{4}{1}$
ONE PAIR	$xxabc$	$\binom{13}{1}\binom{4}{2} \times \binom{12}{3}\binom{4}{1}\binom{4}{1}\binom{4}{1}$
HIGH CARD	$abcde$	$\left(\binom{13}{5} - 10\right)(4^5 - 4)$

HIGH CARD: you begin by choosing five distinct value out of the thirteen. This ensures that we won't get a pair, triple or four of a kind appearing in our hand. We must also make sure that those five values don't form a sequential run (of which there are ten).

So they have to be subtracted, giving us $\binom{13}{5} - 10$. We then have to consider the choices of suits.

There are four choices of suits for each of the five cards, giving us 4^5 possibilities. However, we want to avoid flushes, so they can't all be the same suit (only four ways for that to happen). This gives us $(4^5 - 4)$.

Multiplying them together gives us the required formula.

NOTE: Analyses for the other cases were provided in class.

2. For each situation below, you are betting on the outcome of the roll of two dice.

   In each case, determine the expected value of your bet, and whether or not the bet is favourable.

   Make sure to properly justify your answer.

   (a) If you roll an eleven (i.e. the **sum** of the faces is 11), you win \$9.

   If exactly one of the dice is a two, you lose \$2. Otherwise, no money is exchanged.

   Answer: The probability that you get eleven is $\frac{2}{36}$.

   The probability that you roll exactly one **two** is $2 \times \frac{1}{6} \times \frac{5}{6} = \frac{10}{36}$, since either the first die is a deuce and the second isn't, or vice versa. Note that if this happens, the sum can't also be eleven. So the events are mutually exclusive. This also leaves a $\frac{24}{36} = \frac{2}{3}$ probability that no money is exchanged.

   So applying our formula, we get $E(x) = 9\left(\frac{2}{36}\right) + (-2)\left(\frac{10}{36}\right) + 0\left(\frac{24}{36}\right) = -\frac{1}{18}$, so NOT favourable.

   (b) If the PRODUCT of the numbers on the dice is odd, you win \$3.

   Otherwise (i.e. if the product is even), you lose your \$1.

   Answer: The only way the product of two numbers is odd is if BOTH numbers are odd, so in our case, 1,3 or 5. So the probability that this happens is $\frac{3}{6} \times \frac{3}{6} = \frac{1}{4}$, and the probability that the product is even is $\frac{3}{4}$.

   This gives us an expected value of $E(x) = 3\left(\frac{1}{4}\right) + (-1)\left(\frac{3}{4}\right) = 0$. So the bet is fair, i.e. it's not favourable, but not unfavourable either.

3. In a game of **European** Roulette, you like to bet on lucky number 13. Note that the payoff if you win is the same as for American roulette (i.e. 35 to 1).

   What is the probability of showing a profit after you've bet a dollar on that same number 35 times in a row?

   Given the answer, is this a viable strategy to win at roulette? Explain your answer in detail.

   Answer: To come out on top, you would only need to win AT LEAST once, i.e. you don't lose every time.

   So the probability is simply $1 - \left(\frac{36}{37}\right)^{35} \approx 61.7\%$. Even though you are profitable over half the time, it's not a good strategy because you usually win just a dollar, but when you lose, it's always 35 bucks.

   In fact, we can calculate the expectation pretty easily since it's linear. For us, it means the expectation for all 35 spins is just 35 TIMES the expectation for one spin.

   The expected value of a single bet in European roulette is given by: $(-1)(\frac{36}{37}) + 35(\frac{1}{37}) = \frac{-1}{37} \approx -0.027$.

   So for 35 times, we get $E(x) = 35\left(\frac{-1}{37}\right) = -\frac{35}{37} \approx -0.946$.

   In hindsight (but perhaps only then), it's obviously a bad strategy. If it's unfavourable to bet once, doing it multiple times certainly won't improve your odds, even if "you have a good feeling about it this time."