

1. Given that you have 8 pairs of pants, 3 shirts, and 4 pairs of shoes. How many different outfits can you wear?

$$\text{Answer: } 8 \times 3 \times 4 = 96$$

2. At a rock, paper, and scissors competition, there are 8 competitors. If each person plays exactly one game with every other person, how many games are played in all?

$$\text{Answer: } 7 + 6 + 5 + 4 + 3 + 2 + 1 = \sum_{n=1}^7 n = 28$$

3. Suppose you're only allowed to use the digits 1,2,3, and 4. How many 4-digit numbers can you make under each of the following conditions?

- (a) No other restrictions

$$\text{Answer: } 4^4 = 256$$

- (b) The number must be even

$$\text{Answer: } 4^3 \times 2 = 128$$

- (c) The number must be even and no digits can be re-used

$$\text{Answer: } 2 \times 3! = 12$$

4. From a standard deck, how many 5-card hands will contain exactly 2 hearts and 2 clubs cards?

$$\text{Answer: } \binom{13}{2} \binom{13}{2} \binom{26}{1} = 158\,184$$

5. If you draw two cards (without replacement) from a standard deck, what is the **expected** number of picture cards (jacks, queens or kings) you will have?

$$\text{Answer: } \frac{6}{13}$$

6. There's a race with 10 people. Assuming no ties, how many different ways are there to award the gold, silver, and bronze medals (1st, 2nd and 3rd place)?

$$\text{Answer: } 10 \times 9 \times 8 = 720$$

7. How many different configurations, and games are there for a tic-tac-toe game, when both players have moved exactly twice?

Answer: Configurations: $\binom{9}{2} \binom{7}{2} = 756$ Games: $9 \times 8 \times 7 \times 6 = 3024$

8. On the island of Knights and Knaves, you meet two islanders A and B.

(a) Suppose A says "We're both knaves". What can you say about the islanders?

Answer: A is a knave, and B is a knight.

(b) Suppose A says "B will tell you that I am a knave". What can you say about the islanders?

Answer: B is a knave

9. On the island of Knights and Knaves, you meet two islanders along a path that forks into two. You know that one is a knight and one is a knave, but not which is which. You need to know which of the two paths take you to the village, but can only ask one question to one of the islanders. How do you figure out which path to take?

Answer: Ask either islander "If I were to ask the other islander, which path leads to the village, which would he say?" Then take the other path.

10. You meet three islanders A,B and C. A says "At least one us is a knave." B says "At least one of is a knight." C says "Both A and B are lying." What can you say about the islanders?

Answer: A,B are knights, C is a knave.

11. You have a shuffled packet of cards with 20 cards that has exactly 10 red and 10 black cards. You give Eve SEVEN cards sight unseen. She peeks at her cards and sees that she has x red ones. What does that tell you about the number of black cards you have?

Answer: You have $x + 3$ black cards

12. In the Fitch Cheney poker trick, imagine you have the following 5 cards: 2H, 3S, 5S, 9H, JS. Which cards could be encoded, and for each card, give a suitable encoding?

Answer:

3S encoded: JS, 9H, 2H, 5S

5S encoded: 3S, 2H, JS, 9H

JS encoded: 5S, 9H, 3S, 2H

2H encoded: 9H, JS, 5S, 3S