

**201-NYA-05 - Calculus 1**  
**WORKSHEET: INTEGRALS**

Evaluate the following indefinite integrals:

1.  $\int (4x + 3) dx$
2.  $\int (4x^2 - 8x + 1) dx$
3.  $\int (9t^2 - 4t + 3) dt$
4.  $\int (2t^3 - t^2 + 3t - 7) dt$
5.  $\int \left( \frac{1}{z^3} - \frac{3}{z^2} \right) dz$
6.  $\int \left( \frac{4}{z^7} - \frac{7}{z^4} + z \right) dz$
7.  $\int \left( 3\sqrt{u} + \frac{1}{\sqrt{u}} \right) du$
8.  $\int \left( \sqrt{u^3} - \frac{1}{2}u^{-2} + 5 \right) du$
9.  $\int (2v^{5/4} + 6v^{1/4} + 3v^{-4}) dv$
10.  $\int (3v^5 - v^{5/3}) dv$
11.  $\int (3x - 1)^2 dx$
12.  $\int \left( x - \frac{1}{x} \right)^2 dx$
13.  $\int x(2x + 3) dx$
14.  $\int (2x - 5)(3x + 1) dx$
15.  $\int \frac{8x - 5}{\sqrt[3]{x}} dx$
16.  $\int \frac{2x^2 - x + 3}{\sqrt{x}} dx$
17.  $\int \frac{x^3 - 1}{x - 1} dx$
18.  $\int \frac{x^3 + 3x^2 - 9x - 2}{x - 2} dx$
19.  $\int \frac{(t^2 + 3)^2}{t^6} dt$
20.  $\int \frac{(\sqrt{t} + 2)^2}{t^3} dt$
21.  $\int \frac{3}{4} \cos u du$
22.  $\int -\frac{1}{5} \sin u du$
23.  $\int \frac{7}{\csc x} dx$
24.  $\int \frac{1}{4 \sec x} dx$
25.  $\int (\sqrt{t} + \cos t) dt$
26.  $\int (\sqrt[3]{t^2} - \sin t) dt$
27.  $\int \frac{\sec t}{\cos t} dt$
28.  $\int \frac{1}{\sin^2 t} dt$
29.  $\int (\csc v \cot v \sec v) dv$
30.  $\int (4 + 4 \tan^2 v) dv$
31.  $\int \frac{\sec w \sin w}{\cos w} dw$
32.  $\int \frac{\csc w \cos w}{\sin w} dw$
33.  $\int \frac{(1 + \cot^2 z) \cot z}{\csc z} dz$
34.  $\int \frac{\tan z}{\cos z} dz$
35.  $\int \frac{d}{dx} \sqrt{x^2 + 4} dx$
36.  $\int \frac{d}{dx} \sqrt[3]{x^3 - 8} dx$
37.  $\int \frac{d}{dx} \sin \sqrt[3]{x} dx$
38.  $\int \frac{d}{dx} \sqrt{\tan x} dx$
39.  $\frac{d}{dx} \int x^3 \sqrt{x - 4} dx$
40.  $\frac{d}{dx} \int x^4 \sqrt[3]{x^2 + 9} dx$
41.  $\frac{d}{dx} \int \cot x^3 dx$
42.  $\frac{d}{dx} \int \cos \sqrt{x^2 + 1} dx$

Solve the differential equation subject to the given conditions:

43.  $f'(x) = 12x^2 - 6x + 1$        $f(1) = 5$

44.  $f'(x) = 9x^2 + x - 8$        $f(-1) = 1$

45.  $\frac{dy}{dx} = 4x^{1/2}$        $y = 21$  when  $x = 4$

Evaluate the following definite integrals:

46.  $\int_0^1 2x \, dx$

47.  $\int_2^7 3 \, dv$

48.  $\int_{-1}^0 (x - 2) \, dx$

49.  $\int_2^5 (-3v + 4) \, dv$

50.  $\int_{-1}^1 (t^2 - 2) \, dt$

51.  $\int_0^3 (3x^2 + x - 2) \, dx$

52.  $\int_0^1 (2t - 1)^2 \, dt$

53.  $\int_{-1}^1 (t^3 - 9t) \, dt$

54.  $\int_1^2 \left( \frac{3}{x^2} - 1 \right) \, dx$

55.  $\int_{-2}^{-1} \left( u - \frac{1}{u^2} \right) \, du$

56.  $\int_1^4 \frac{u - 2}{\sqrt{u}} \, du$

57.  $\int_{-3}^3 v^{1/3} \, dv$

58.  $\int_{-1}^1 (\sqrt[3]{t} - 2) \, dt$

59.  $\int_1^8 \sqrt{\frac{2}{x}} \, dx$

60.  $\int_0^1 \frac{x - \sqrt{x}}{3} \, dx$

61.  $\int_0^2 (2 - t)\sqrt{t} \, dt$

62.  $\int_{-1}^0 (t^{1/3} - t^{2/3}) \, dt$

63.  $\int_{-8}^{-1} \frac{x - x^2}{2\sqrt[3]{x}} \, dx$

64.  $\int_0^3 |2x - 3| \, dx$

65.  $\int_0^4 |x^2 - 4x + 3| \, dx$

66.  $\int_0^\pi (1 + \sin x) \, dx$

67.  $\int_0^{\pi/4} \frac{1 - \sin^2 \theta}{\cos^2 \theta} \, d\theta$

68.  $\int_{-\pi/6}^{\pi/6} \sec^2 x \, dx$

69.  $\int_{\pi/4}^{\pi/2} (2 - \csc^2 x) \, dx$

70.  $\int_{-\pi/3}^{\pi/3} 4 \sec \theta \tan \theta \, d\theta$

71.  $\int_{-\pi/2}^{\pi/2} (2t + \cos t) \, dt$

72.  $\int_1^e \left( 2x + \frac{1}{x} \right) \, dx$

73.  $\int_1^5 \frac{x + 1}{x} \, dx$

74.  $\int_0^2 (e^x + 6) \, dx$

75.  $\int_0^3 (t - e^t) \, dt$

76.  $\int_{-1}^1 (e^\theta + \sin \theta) \, d\theta$

77.  $\int_e^{2e} \left( \cos x - \frac{1}{x} \right) \, dx$

## ANSWERS

### Indefinite integrals:

- |                                                                 |                                                              |                                       |
|-----------------------------------------------------------------|--------------------------------------------------------------|---------------------------------------|
| 1. $2x^2 + 3x + C$                                              | 14. $2x^3 - \frac{13x^2}{2} - 5x + C$                        | 26. $\frac{3t^{5/3}}{5} + \cos t + C$ |
| 2. $\frac{4x^3}{3} - 4x^2 + x + C$                              | 15. $\frac{24x^{5/3}}{5} - \frac{15x^{2/3}}{2} + C$          | 27. $\tan t + C$                      |
| 3. $3t^3 - 2t^2 + 3t + C$                                       | 16. $\frac{4x^{5/2}}{5} - \frac{2x^{3/2}}{3} + 6x^{1/2} + C$ | 28. $-\cot t + C$                     |
| 4. $\frac{t^4}{2} - \frac{t^3}{3} + \frac{3t^2}{2} - 7t + C$    | 17. $\frac{x^3}{3} + \frac{x^2}{2} + x + C$                  | 29. $-\cot v + C$                     |
| 5. $-\frac{z^{-2}}{2} + 3z^{-1} + C$                            | 18. $\frac{x^3}{3} + \frac{5x^2}{2} + x + C$                 | 30. $4 \tan v + C$                    |
| 6. $-\frac{4z^{-6}}{6} + \frac{7z^{-3}}{3} + \frac{z^2}{2} + C$ | 19. $-t^{-1} - 2t^{-3} - \frac{9t^{-5}}{5} + C$              | 31. $\sec w + C$                      |
| 7. $2u^{3/2} + 2u^{1/2} + C$                                    | 20. $-t^{-1} - \frac{8t^{-3/2}}{3} - 2t^{-2} + C$            | 32. $-\csc w + C$                     |
| 8. $\frac{2u^{5/2}}{5} + \frac{u^{-1}}{2} + 5u + C$             | 21. $\frac{3}{4} \sin u + C$                                 | 33. $-\csc z + C$                     |
| 9. $\frac{8v^{9/4}}{9} + \frac{24v^{5/4}}{5} - v^{-3} + C$      | 22. $\frac{1}{5} \cos u + C$                                 | 34. $\sec z + C$                      |
| 10. $\frac{v^6}{2} - \frac{3v^{8/3}}{8} + C$                    | 23. $-7 \cos x + C$                                          | 35. $\sqrt{x^2 + 4} + C$              |
| 11. $3x^3 - 3x^2 + x + C$                                       | 24. $\frac{1}{4} \sin x + C$                                 | 36. $\sqrt[3]{x^3 - 8} + C$           |
| 12. $\frac{x^3}{3} - 2x - x^{-1} + C$                           | 25. $\frac{2t^{3/2}}{3} + \sin t + C$                        | 37. $\sin \sqrt[3]{x} + C$            |
| 13. $\frac{2x^3}{3} + \frac{3x^2}{2} + C$                       |                                                              | 38. $\sqrt{\tan x} + C$               |

### Differential equations:

43.  $f(x) = 4x^3 - 3x^2 + x + 3$   
44.  $f(x) = 3x^3 + \frac{x^2}{2} - 8x - \frac{9}{2}$   
45.  $y(x) = \frac{8x^{3/2}}{3} - \frac{1}{3}$

### Definite integrals:

- |                     |                             |                          |                                   |
|---------------------|-----------------------------|--------------------------|-----------------------------------|
| 46. 1               | 54. $\frac{1}{2}$           | 62. $\frac{-27}{20}$     | 70. 0                             |
| 47. 15              | 55. -2                      | 63. $\frac{1523}{20}$    | 71. 2                             |
| 48. $-\frac{5}{2}$  | 56. $\frac{2}{3}$           | 64. $\frac{9}{2}$        | 72. $e^2$                         |
| 49. $-\frac{39}{2}$ | 57. 0                       | 65. 4                    | 73. $4 + \ln 5$                   |
| 50. $-\frac{10}{3}$ | 58. -4                      | 66. $\pi + 2$            | 74. $e^2 + 11$                    |
| 51. $\frac{51}{2}$  | 59. $2(4 - \sqrt{2})$       | 67. $\frac{\pi}{4}$      | 75. $\frac{11}{2} - e^3$          |
| 52. $\frac{1}{3}$   | 60. $-\frac{1}{18}$         | 68. $\frac{2}{\sqrt{3}}$ | 76. $e - e^{-1}$                  |
| 53. 0               | 61. $\frac{16\sqrt{2}}{15}$ | 69. $\frac{\pi}{2} - 1$  | 77. $\sin(2e) - \sin(e) - \ln(2)$ |