

Integration of all kinds

For more problems see Stewart §7.5

1. $\int \frac{\cos(\frac{1}{x}) - x}{x^4} dx$
 2. $\int \frac{1}{e^x - e^{-x}} dx$
 3. $\int \tan^4(\theta) d\theta$
 4. $\int \frac{1}{\sqrt{x+1} + \sqrt{x}} dx$
 5. $\int \frac{3x^2 - 2}{x^2 - 2x - 8} dx$
 6. $\int \sqrt{\frac{1-x}{1+x}} dx$
 7. $\int \frac{3x-1}{\sqrt{4x^2-1}} dx$
 8. $\int_1^3 \frac{1}{\sqrt{15+2x-x^2}} dx$
 9. $\int \frac{1}{x\sqrt{4x+1}} dx$
 10. $\int \frac{x^2+2x-5}{x\sqrt{9x^2-4}} dx$
 11. $\int \frac{1-\sqrt{x^2-4}}{x^3\sqrt{x^2-4}} dx$
 12. $\int_0^\pi \sqrt{\cos(\theta)+1} d\theta$
 13. $\int \frac{\arctan(y)}{y^2} dy$
 14. $\int \frac{\ln(\tan(x))}{\sin(2x)} dx$
 15. $\int \frac{x + \arccos(2x)}{\sqrt{1-4x^2}} dx$
 16. $\int \frac{\sqrt{4x^2-9}}{x} dx$
 17. $\int \frac{e^{1/v}}{v^6} dv$
 18. $\int \frac{e^x}{(e^{2x}-9)^{3/2}} dx$
 19. $\int \frac{1}{x \ln^2(5x)} dx$
 20. $\int \frac{x^2+3x+1}{(2x^2+1)(x-3)} dx$
 21. $\int \frac{e^{3x}}{e^{2x}-4} dx$
 22. $\int \frac{1}{(4x^2+1)^3} dx$
 23. $\int \frac{1}{\sqrt{x}\sqrt{x-3}} dx$
 24. $\int \frac{3\sqrt[4]{x}}{\sqrt{x}+3\sqrt[3]{x}} dx$
 25. $\int \cos^3(x) dx$
 26. $\int \frac{1}{x \ln^2(x) + x} dx$
 27. $\int \frac{1}{(x^2+2x+10)^{5/2}} dx$
 28. $\int_0^1 \arctan(\sqrt{x}) dx$
 29. $\int_{\pi/4}^{\pi/3} \frac{x}{\sin^2(x)} dx$
 30. $\int_{\pi/6}^{\pi/4} \sin(x) \cos(2x) dx$
 31. $\int \frac{x \ln(x)}{(x^2-1)^{2/3}} dx$
 32. $\int \sin(2x) \arctan(\sin(x)) dx$
 33. $\int \frac{1}{\sqrt{15+6x-9x^2}} dx$
 34. $\int \frac{e^x}{e^{2x}+4e^x+5} dx$
 35. $\int \frac{\ln(x)}{x\sqrt[3]{2+\ln(x)}} dx$
 36. $\int (2x^3-x+4)e^{\frac{x-3}{2}} dx$
 37. $\int \frac{5-2x^3}{\sqrt{3x-7}} dx$
 38. $\int_0^{\pi^3/27} \cos(\sqrt[3]{x}) dx$
 39. $\int x^2 \arccos(x) dx$
 40. $\int \sqrt{x} \ln(x+1) dx$
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Answers

1. $-\frac{\sin(1/x)}{x^2} - \frac{2\cos(1/x)}{x} + 2\sin(1/x) + \frac{1}{2x^2} + C$
2. $\frac{1}{2} \ln \left| \frac{e^x - 1}{e^x + 1} \right| + C$
3. $\frac{1}{3} \tan^3(\theta) - \tan(\theta) + \theta + C$
4. $\frac{2}{3}(x+1)^{3/2} - \frac{2}{3}x^{3/2} + C$
5. $3x - \frac{5}{3} \ln|x+2| + \frac{23}{3} \ln|x-4| + C$
6. $\arcsin(x) + \sqrt{1-x^2} + C$
7. $\frac{3}{4}\sqrt{4x^2-1} - \frac{1}{2} \ln|2x + \sqrt{4x^2-1}| + C$
8. $\arcsin\left(\frac{x-1}{4}\right)\Big|_1^3 = \frac{\pi}{6}$
9. $\ln \left| \frac{\sqrt{4x+1}-1}{\sqrt{4x+1}+1} \right| + C$

10. $\frac{1}{9}\sqrt{9x^2-4} + \frac{2}{3}\ln|3x + \sqrt{9x^2-4}| - \frac{5}{2}\operatorname{arcsec}\left(\frac{3x}{2}\right) + C$
11. $\frac{1}{16}\operatorname{arcsec}\left(\frac{x}{2}\right) + \frac{\sqrt{x^2-4}+4}{8x^2} + C$
12. $2\sqrt{1-\cos(x)} + C$, wherever $\sin(x) \geq 0$. $2\sqrt{1-\cos(x)}\Big|_0^\pi = 2\sqrt{2}$
13. $-\frac{\arctan(y)}{y} + \ln|y| - \frac{1}{2}\ln|y^2+1| + C$
14. $\frac{1}{4}\ln^2|\tan(x)| + C$
15. $-\frac{1}{4}\sqrt{1-4x^2} - \frac{1}{4}\arccos^2(2x) + C$
16. $\sqrt{4x^2-9} - 3\arctan\left(\frac{\sqrt{4x^2-9}}{3}\right) + C$ OR $\sqrt{4x^2-9} - 3\operatorname{arcsec}\left(\frac{2x}{3}\right) + C$
17. $-e^{1/v}\left(\frac{1}{v^4} - \frac{4}{v^3} + \frac{12}{v^2} - \frac{24}{v} + 24\right) + C$
18. $-\frac{e^x}{9\sqrt{e^{2x}-9}} + C$
19. $-\frac{1}{\ln(5x)} + C$
20. $-\frac{1}{4}\ln|2x^2+1| + \ln|x-3| + C$
21. $e^x + \ln\left|\frac{e^x-2}{e^x+2}\right| + C$
22. $\frac{3}{16}\arctan(2x) + \frac{3x}{8(4x^2+1)} + \frac{x}{4(4x^2+1)^2} + C$ OR $\frac{3}{16}\arctan(2x) + \frac{x}{2(4x^2+1)} + \frac{x(1-4x^2)}{8(4x^2+1)^2}$
23. $\ln|2x-3 + \sqrt{(2x-3)^2-9}| + C$ OR $2\ln|\sqrt{x-3} + \sqrt{x}|$
24. $4x^{3/4} - \frac{108}{7}x^{7/12} + \frac{324}{5}x^{5/12} - 324x^{1/4} + 2916x^{1/12} - 2916\sqrt{3}\arctan\left(\frac{x^{1/12}}{\sqrt{3}}\right) + C$
25. $\sin(x) - \frac{1}{3}\sin^3(x) + C$
26. $\arctan(\ln(x)) + C$
27. $\frac{1}{81}\left[\frac{x+1}{\sqrt{x^2+2x+10}} - \frac{1}{3}\frac{(x+1)^3}{(x^2+2x+10)^{3/2}}\right] + C$
28. $[x\arctan(\sqrt{x}) - \sqrt{x} + \arctan(\sqrt{x})]\Big|_0^1 = \frac{\pi}{2} - 1$
29. $[-x\cot(x) + \ln|\sin(x)]\Big|_{\pi/4}^{\pi/3} = \frac{\pi}{4} - \frac{\pi}{3\sqrt{3}} + \frac{1}{2}\ln\left|\frac{3}{2}\right|$
30. $\left[-\frac{2}{3}\cos^3(x) + \cos(x)\right]\Big|_{\pi/6}^{\pi/4}$ OR $\left[\frac{2}{3}\sin(x)\sin(2x) + \frac{1}{3}\cos(x)\cos(2x)\right]\Big|_{\pi/6}^{\pi/4} = \frac{\sqrt{2}}{3} - \frac{\sqrt{3}}{4}$
31. $\frac{3}{2}\ln(x)\sqrt[3]{x^2-1} - \frac{9}{4}\left[\sqrt[3]{x^2-1} - \frac{1}{3}\ln|\sqrt[3]{x^2-1}+1| - \frac{2}{3}\ln|(x^2-1)^{2/3} - \sqrt[3]{x^2-1}+1|\right] - \frac{3}{\sqrt{3}}\arctan\left(\frac{2\sqrt[3]{x^2-1}-1}{\sqrt{3}}\right) + C$
32. $\sin^2(x)\arctan(\sin(x)) - \sin(x) + \arctan(\sin(x)) + C$
33. $\frac{1}{3}\arcsin\left(\frac{3x-1}{4}\right) + C$
34. $\arctan(e^x+2) + C$
35. $\frac{3}{5}(\ln(x)+2)^{5/3} - 3(\ln(x)+2)^{2/3} + C$
36. $2e^{\frac{x-3}{2}}[(2x^3-x+4) - 2(6x^2-1) + 48x - 96] + C$
37. $\frac{2}{3}(5-2x^3)(3x-7)^{1/2} + \frac{8}{9}x^2(3x-7)^{3/2} - \frac{32}{135} + \frac{64}{2835}(3x-7)^{7/2} + C$
38. $3\sqrt[3]{x^2}\sin(\sqrt[3]{x}) + 6\sqrt[3]{x}\cos(\sqrt[3]{x}) - 6\sin(\sqrt[3]{x})\Big|_0^{\pi^{3/27}} = \frac{\sqrt{3}\pi^2}{6} + \pi - 3\sqrt{3}$
39. $\frac{1}{3}x^3\arccos(x) - \frac{1}{9}x^2\sqrt{1-x^2} - \frac{2}{9}\sqrt{1-x^2} + C$
40. $\frac{2}{3}x^{3/2}\ln(x+1) - \frac{4}{9}x^{3/2} + \frac{4}{3}\sqrt{x} - \frac{4}{3}\arctan(\sqrt{x}) + C$