Integration of all kinds

For more problems see Stewart §7.5

1. $\int \dfrac{\cos\left(\frac{1}{x}\right) - x}{x^4}\, dx$ *SPLIT* *subr parts*

2. $\int \dfrac{1}{e^x - e^{-x}}\, dx$ *Sub $u = e^x$*

3. $\int \tan^4(\theta)\, d\theta$ *trig id*

4. $\int \dfrac{1}{\sqrt{x+1} + \sqrt{x}}\, dx$ *rationalize*

5. $\int \dfrac{3x^2 - 2}{x^2 - 2x - 8}\, dx$ *LD PF*

6. $\int \sqrt{\dfrac{1-x}{1+x}}\, dx$ $\frac{\sqrt{1-x}}{\sqrt{1-x}}$, *split*

7. $\int \dfrac{3x - 1}{\sqrt{4x^2 - 1}}\, dx$ *split, trig sub*

8. $\int_1^3 \dfrac{1}{\sqrt{15 + 2x - x^2}}\, dx$ *inv trig*

9. $\int \dfrac{1}{x\sqrt{4x + 1}}\, dx$ $u = \sqrt{4x+1}$

10. $\int \dfrac{x^2 + 2x - 5}{x\sqrt{9x^2 - 4}}\, dx$ *split TRIGSUB*

11. $\int \dfrac{1 - \sqrt{x^2 - 4}}{x^3\sqrt{x^2 - 4}}\, dx$ *TRIG SUB*

12. $\int_0^\pi \sqrt{\cos(\theta) + 1}\, d\theta$ *conjugate*

13. $\int \dfrac{\arctan(y)}{y^2}\, dy$ *parts*

14. $\int \dfrac{\ln(\tan(x))}{\sin(2x)}\, dx$ *sub $u = \ln(\tan x)$*

15. $\int \dfrac{x + \arccos(2x)}{\sqrt{1 - 4x^2}}\, dx$ *split + parts*

16. $\int \dfrac{\sqrt{4x^2 - 9}}{x}\, dx$ *TRIG SUB*

17. $\int \dfrac{e^{1/v}}{v^6}\, dv$ $u = \frac{1}{v}$ *parts*

18. $\int \dfrac{e^x}{(e^{2x} - 9)^{3/2}}\, dx$ $u = e^x$

19. $\int \dfrac{1}{x \ln^2(5x)}\, dx$ $u = \ln(5x)$

20. $\int \dfrac{x^2 + 3x + 1}{(2x^2 + 1)(x - 3)}\, dx$ *PF*

21. $\int \dfrac{e^{3x}}{e^{2x} - 4}\, dx$ $u = e^x$

22. $\int \dfrac{1}{(4x^2 + 1)^3}\, dx$ *TRIG SUB*

23. $\int \dfrac{1}{\sqrt{x}\sqrt{x - 3}}\, dx$ $\frac{1}{\sqrt{x^2-3x}}$ *TRIG AP*

24. $\int \dfrac{3\sqrt[4]{x}}{\sqrt{x} + 3\sqrt[3]{x}}\, dx$ $u = x^{1/12}$

25. $\int \cos^3(x)\, dx$ *easy*

26. $\int \dfrac{1}{x \ln^2(x) + x}\, dx$ *Factor $\frac{1}{x}(\ln^2 x + 1)$*

27. $\int \dfrac{1}{(x^2 + 2x + 10)^{5/2}}\, dx$ *TRIGSUB*

28. $\int_0^1 \arctan(\sqrt{x})\, dx$ *sub + parts*

29. $\int_{\pi/4}^{\pi/3} \dfrac{x}{\sin^2(x)}\, dx$ $x \csc^2 x$ *parts*

30. $\int_{\pi/6}^{\pi/4} \sin(x)\cos(2x)\, dx$ *ID or par* *$\cos^2 x - \sin^2 x$ bou*

31. $\int \dfrac{x \ln(x)}{(x^2 - 1)^{2/3}}\, dx$ *parts + TRIGSUB?*

32. $\int \sin(2x)\arctan(\sin(x))\, dx$ $u = \sin x$ *id parts*

33. $\int \dfrac{1}{\sqrt{15 + 6x - 9x^2}}\, dx$ *inv trig*

34. $\int \dfrac{e^x}{e^{2x} + 4e^x + 5}\, dx$ $u = e^x$

35. $\int \dfrac{\ln(x)}{x\sqrt[3]{2 + \ln(x)}}\, dx$ $u = \ln x + 2$

36. $\int (2x^3 - x + 4)e^{\frac{x-3}{2}}\, dx$ *parts*

37. $\int \dfrac{5 - 2x^3}{\sqrt{3x - 7}}\, dx$ $u = \sqrt{3x-7}$

38. $\int_0^{\pi^3/27} \cos(\sqrt[3]{x})\, dx$ $u = \sqrt[3]{x}$ *parts*

39. $\int x^2 \arccos(x)\, dx$ *parts*

40. $\int \sqrt{x}\ln(x + 1)\, dx$ *parts*

Answers

1. $-\dfrac{\sin(1/x)}{x^2} - \dfrac{2\cos(1/x)}{x} + 2\sin(1/x) + \dfrac{1}{2x^2} + C$

2. $\dfrac{1}{2}\ln\left|\dfrac{e^x - 1}{e^x + 1}\right| + C$

3. $\dfrac{1}{3}\tan^3(\theta) - \tan(\theta) + \theta + C$

4. $\dfrac{2}{3}(x + 1)^{3/2} - \dfrac{2}{3}x^{3/2} + C$

5. $3x - \frac{5}{3}\ln|x + 2| + \frac{23}{3}\ln|x - 4| + C$

6. $\arcsin(x) + \sqrt{1 - x^2} + C$

7. $\frac{3}{4}\sqrt{4x^2 - 1} - \frac{1}{2}\ln\left|2x + \sqrt{4x^2 - 1}\right| + C$

8. $\arcsin\left(\frac{x-1}{4}\right)\Big|_1^3 = \dfrac{\pi}{6}$

9. $\ln\left|\dfrac{\sqrt{4x + 1} - 1}{\sqrt{4x + 1} + 1}\right| + C$