201-NYB-05 - Calculus 2

REVIEW WORKSHEET FOR TEST #2

1. Evaluate the following limits. Write DNE, $-\infty$ or ∞ if appropriate.

   (a) $\displaystyle\lim_{x\to\infty} x\left(e^{3/x}-1\right)$

   (b) $\displaystyle\lim_{x\to 0} \frac{\arctan(x)}{\tan(2x)}$

   (c) $\displaystyle\lim_{\theta\to 0} \frac{1-\cos(4\theta)-2\theta^2}{\sin^2(5\theta)}$

   (d) $\displaystyle\lim_{x\to 1^+} \ln(1-x)-\ln(1-x^2)$

   (e) $\displaystyle\lim_{x\to 3^+} \frac{1}{\ln(x-2)}-\frac{1}{x-3}$

   (f) $\displaystyle\lim_{x\to\infty} \left(1+\frac{6}{x}+\frac{1}{x^2}\right)^x$

2. Determine whether each improper integral is convergent or divergent. If it converges, give its value. *Use correct notation throughout.*

   (a) $\displaystyle\int_0^{\pi/2} \frac{1}{\cos(x)}\,dx$

   (b) $\displaystyle\int_2^{\infty} \frac{1}{\sqrt[4]{3x-6}}\,dx$

   (c) $\displaystyle\int_0^{\pi/4} \frac{\sec^2(x)}{\sqrt{\tan(x)}}\,dx$

   (d) $\displaystyle\int_4^{6} \frac{x}{x-5}\,dx$

   (e) $\displaystyle\int_2^{4} \frac{1}{x\sqrt{x^2-4}}\,dx$

   (f) $\displaystyle\int_2^{3} \ln(x-2)\,dx$

3. Find the area enclosed between the two curves.

   (a) $y=x^3-2x,\quad y=3x$ (b) $x=5-y^2,\quad y=2x$

4. Consider the region enclosed by the three curves:

   $$y=1,\qquad y=x+1,\qquad \text{and } y=2-\frac{x}{2}.$$

   (a) Find the volume of the solid obtained by rotating this area around the x-axis.

   (b) Find the volume of the solid obtained by rotating this area around the axis $x=2$.

5. Consider the region enclosed by the two curves $y=x^3$ and $y=2x-x^2$. Express each volume as a definite integral **without evaluating it**.

   (a) The volume obtained by rotating the region around the axis $y=2$.

   (b) The volume obtained by rotating the region around the axis $x=-2$.

6. Find the arc length of each function between the points provided:

   (a) $y=13+4x\sqrt{x}$, from $x=0$ to $x=1$,

   (b) $x=\dfrac{2y^{3/2}}{3}-\dfrac{y^{1/2}}{2}$, from $y=1$ to $y=4$,

7. Find the particular solution of each differential equation.

   (a) $y'=\dfrac{\sin(2x)}{\cos(3y)}$, with $y(\pi/2)=\pi/3$

   (b) $x+ye^{2x}\dfrac{dy}{dx}=0$, with $y(0)=-1$

   (c) $y'=\dfrac{y}{x^2-2x}$, with $y(1)=3$

8. A chemical reaction gradually transforms a compound A into another compound B. Suppose you have initially 20 grams of compound A, and that there is 18 grams left after 1 minute.

   (a) Suppose that the rate of reaction is proportional to the amount of compound A remaining at that time. Set up and solve the corresponding differential equation.

   (b) Suppose instead that the rate of reaction is proportional to the *square* of the amount of compound A remaining. Set up and solve the corresponding differential equation.

   (c) Find what happens to the amount of compound A if you wait a long time. Is it the same behaviour for both of the above models?

ANSWERS:

1. (a) 3 (b) $\frac{1}{2}$ (c) $\frac{6}{25}$ (d) $\ln(1/2) = -\ln(2)$ (e) $\frac{1}{2}$ (f) e^6

2. (a) ∞, divergent (b) ∞, divergent (c) 2, convergent
   (d) divergent (e) $\frac{\pi}{6}$, convergent (f) -1, convergent

3. (a) $\displaystyle\int_{-\sqrt{5}}^{0}(x^3-5x)\,dx + \int_{0}^{\sqrt{5}}(5x-x^3)\,dx = \frac{25}{2}$
   (b) $\displaystyle\int_{-5/2}^{2}\left(5-y^2-\frac{y}{2}\right)\,dy = \frac{243}{16}$

4. (a) $\displaystyle\int_{1}^{5/3} 2\pi y(4-2y-y+1)\,dy = \frac{44\pi}{27}$
   (b) $\displaystyle\int_{1}^{5/3}\left[\pi(2-y+1)^2-\pi(2-4+2y)^2\right]\,dy = \frac{40\pi}{27}$

5. (a) $\displaystyle\int_{-2}^{0}\left[\,\pi(2-2x+x^2)^2-\pi(2-x^3)^2\,\right]\,dx + \int_{0}^{1}\left[\,\pi(2-x^3)^2-\pi(2-2x+x^2)^2\,\right]\,dx$
   (b) $\displaystyle\int_{-2}^{0} 2\pi(x+2)(x^3-2x+x^2)\,dx + \int_{0}^{1} 2\pi(x+2)(2x-x^2-x^3)\,dx$

6. (a) $\dfrac{37^{3/2}-1}{54}$ (b) $\dfrac{31}{6}$

7. (a) $y = \dfrac{1}{3}\sin^{-1}\left(-\dfrac{3}{2}(1+\cos(2x))\right)$
   (b) $y = -\sqrt{\dfrac{e^{-2x}}{2}(2x+1)+\dfrac{1}{2}}$
   (c) $y = 3\sqrt{\left|\dfrac{x-2}{x}\right|}$

8. (a) $\dfrac{dA}{dt} = kA \quad\Rightarrow\quad A(t) = 20\left(\dfrac{9}{10}\right)^t$
   (b) $\dfrac{dA}{dt} = kA^2 \quad\Rightarrow\quad A(t) = \dfrac{180}{t+9}$
   (c) in both cases, the amount of compound A goes to 0, $\lim\limits_{t\to\infty} A(t) = 0$