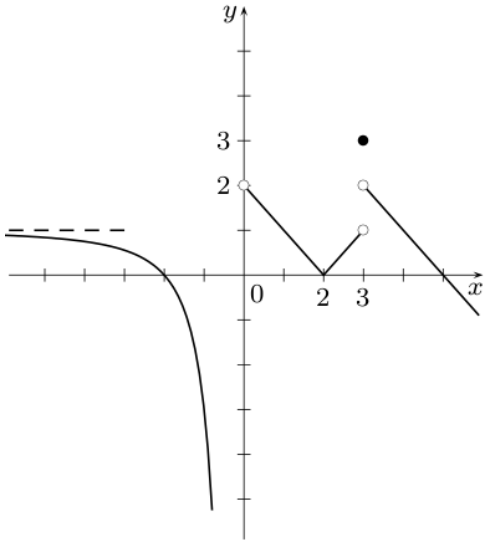
201-103-RE - Calculus 1

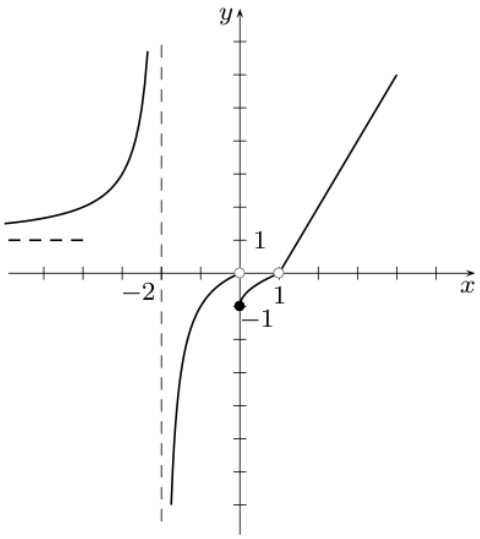
WORKSHEET: CONTINUITY

1. For each graph, determine where the function is **discontinuous**. Justify for each point by: (i) saying which condition fails in the definition of continuity, and (ii) by mentioning which type of discontinuity it is.

(a)



(b)



2. For each function, determine the interval(s) of continuity.

(a) $f(x) = x^2 + e^x$

(b) $f(x) = \dfrac{3x+1}{2x^2-3x-2}$

(c) $f(x) = \sqrt[4]{5-x}$

(d)* $f(x) = \dfrac{2}{4-x^2} + \dfrac{1}{\sqrt{x^2-x-12}}$

3. For each piecewise defined function, determine where $f(x)$ is continuous (or where it is discontinuous). Justify your answer in detail.

(a) $f(x) = \begin{cases} 2^x - 3x^2 & \text{for } x \le 1 \\ \log_{10}(x) + x & \text{for } x > 1 \end{cases}$

(b) $f(x) = \begin{cases} \frac{2x}{3-x} & \text{for } x \le 0 \\ x^2 - 3x & \text{for } 0 < x < 2 \\ \frac{x^2-8}{x} & \text{for } x > 2 \end{cases}$

4. Find all the value(s) of the parameter c (if possible), to make the given function continuous everywhere.

(a) $f(x) = \begin{cases} c \cdot 3^x - x^2 + 2c & \text{for } x \le 0 \\ 2x^5 + c(x+1) + 16 & \text{for } x > 0 \end{cases}$

(b) $$f(x) = \begin{cases} 2(cx)^3 + x - 1 & \text{for } x \leq 1 \\ 2cx + (x-1)^2 & \text{for } x > 1 \end{cases}$$

(c) $$f(x) = \begin{cases} 3x + c & \text{for } x < -1 \\ x^2 - c & \text{for } -1 \leq x \leq 2 \\ 3 & \text{for } x > 2 \end{cases}$$

5.* Consider the function $f(x) = \lfloor x \rfloor$, the greatest integer function (also called the floor function or the step function). Where is this function discontinuous?

6.* Find an example of a function such that the limit exists at every x, but that has an infinite number of discontinuities. (You can describe the function and/or write a formula down and/or draw a graph.)

PARTIAL ANSWERS:

1. (a) $x = 0, 3$ (b) $x = -2, 0, 1$
2. (a) $\mathbb{R}$ (b) $\mathbb{R}\backslash\{-1/2, 2\}$ (c) $(-\infty, 5]$ (d) $(-3, 2) \cup (-2, 2) \cup (2, 4)$
3. (a) discontinuous only at $x = 1$ (b) discontinuous only at $x = 2$
4. (a) $c = 8$ (b) $c = -1, 0, 1$ (c) no solution possible
5. discontinuous at every integer, $x = \ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$
6. many answers are possible, show me your solution!