201-103-RE - Calculus 1

WORKSHEET: DERIVATIVES (PART 1)

1. Find the derivative of each function, by using **the constant rule and the power rule**. DO NOT SIMPLIFY YOUR ANSWERS.

   (a) $f(x) = 10x^{2.1} + 365$

   (b) $f(r) = \frac{4}{3}\pi r^3$

   (c) $f(x) = -x^3 + 2x^2 - 6$

   (d) $f(t) = \frac{4}{t^4} - \frac{3}{t^3} + \frac{2}{t}$

   (e) $f(x) = \sqrt{x^3} - 3x^{-2} + 2x^{-\frac{12}{13}}$

   (f) $f(x) = \frac{3x - 5x^3}{\sqrt[3]{x}}$

   (g) $f(z) = \sqrt{5}\,z + \sqrt{11z}$

   (h) $f(x) = (3x^2 - 5)(x^{-2} + 2x^5)$

   (i) $f(x) = \frac{(3x+1)x}{\sqrt{x}}$

2. Find the derivative of each function. The focus is now on **the trigonometric functions, and the product/quotient rules**. DO NOT SIMPLIFY YOUR ANSWERS.

   (a) $f(x) = 3\tan(x) - \sec(x)$

   (b) $f(x) = \frac{-1}{\sin(x)} + 5\cot(x)$

   (c) $f(x) = (3 - 6x^4)(\sin(x) - 1)$

   (d) $f(x) = (1 + 2x + 3x^2)(5x^5 - 4x^4)$

   (e) $f(t) = \left(\frac{1}{t^3} - \frac{1}{t}\right)\cos(t)$

   (f) $f(\theta) = \theta\cot(\theta) - \theta^2\cos(\theta)$

   (g) $f(x) = \frac{x^3 + 1}{3 - x}$

   (h) $f(t) = \frac{1}{1 - 4t^{-2}}$

   (i) $f(x) = \frac{x}{x^2 - 4} - \frac{x - 1}{x^2 + 4}$

   (j) $f(y) = \frac{\sqrt{y} - \sin(y)}{\tan(y) + y^5}$

   (k) $f(x) = \frac{x}{x + 3/x}$

   (l) $f(x) = \frac{x^2\cot(x)}{x + 1}$

   (m) $f(x) = x\sin(x) + \frac{x}{\cos(x)}$

   (n) $f(x) = \frac{\sqrt{x^3}}{\sec(x)(2x + 3)}$

   (o) $f(t) = \frac{t^2 + 1}{(2t - 3)^2}$

3. Find the derivative of the following functions, by using **the chain rule**. DO NOT SIMPLIFY YOUR ANSWERS.

   (a) $f(x) = \sqrt{x^2 - 3x^{-1} + 5}$

   (b) $f(x) = (2\sin(x) - 3x^9)^{25}$

   (c) $f(x) = \csc(3x - 1)$

   (d) $f(r) = \pi\tan(\pi r^2 - 5r)$

   (e) $f(x) = \left(1 + (2x + 1)^6\right)^7$

   (f) $f(x) = \sin^3(x^2)$

   (g) $f(x) = \sqrt{\cos(x^2 - 3x^{-8} + 1)}$

   (h) $f(x) = \tan(\cot(x^2))$

   (i) $f(x) = \sin^2(\cos^2(x))$

   (j) $f(x) = \sec(3x)\csc(5x)$

   (k) $f(x) = (3x^2 - 9)^{17}(x^4 - x^2 + 1)^{31}$

   (l) $f(x) = \frac{\sqrt{x^2 + 1}}{\sqrt[3]{x^2 - 1}}$

   (m) $f(x) = \frac{\sin(x^2)}{\cos^2(x)}$

   (n) $f(x) = (x^3 - 8)\tan^2(5x - 3)$

   (o) $f(x) = \frac{(3x^3 + x)^4}{(\sqrt{x} - x^{-2})^5}$

   (p) $f(x) = \left(\frac{6x - x^4}{\sin(x)}\right)^6$

4. Let $f(x), g(x), h(x)$ be functions that satisfy the following conditions:

$$f(1) = 3, f'(1) = -4, f'(-1) = 5, g(1) = -1, g'(1) = 3, h(1) = 4, \text{ and } h'(1) = 5$$

Compute $y'(1)$ for each of the following:

(a) $y = (f(x) + g(x))^3$

(b) $y = \dfrac{f(x)g(x)}{h(x)}$

(c) $y = f(x)g(x)h(x)$

(d) $y = (f(g(x)))^2$

ANSWERS:

1. Power rule

(a) $f'(x) = 21x^{1.1}$

(b) $f'(r) = 4\pi r^2$

(c) $f'(x) = -3x^2 + 4x$

(d) $f'(t) = -16t^{-5} + 9t^{-4} - 2t^{-2}$

(e) $f'(x) = \dfrac{3}{2}x^{1/2} + 6x^{-3} - \dfrac{24}{13}x^{-25/13}$

(f) $f'(x) = 2x^{-1/3} - \dfrac{40}{3}x^{5/3}$

(g) $f'(z) = \sqrt{5} + \dfrac{\sqrt{11}}{2}z^{-1/2}$

(h) $f'(x) = 42x^6 + 10x^{-3} - 50x^4$

(i) $f'(x) = \dfrac{9}{2}x^{1/2} + \dfrac{1}{2}x^{-1/2}$

2. Product/quotient rules

(a) $f'(x) = 3\sec^2(x) - \sec(x)\tan(x)$

(b) $f'(x) = \csc(x)\cot(x) - 5\csc^2(x)$

(c) $f'(x) = -24x^3(\sin(x) - 1) + (3 - 6x^4)\cos(x)$

(d) $f'(x) = (2 + 6x)(5x^5 - 4x^4) + (1 + 2x + 3x^2)(25x^4 - 16x^3)$

(e) $f'(t) = (-3t^{-4} + t^{-2})\cos(t) + (t^{-3} - t^{-1})(-\sin(t))$

(f) $f'(\theta) = \cot(\theta) + \theta(-\csc^2(\theta)) - 2\theta\cos(\theta) + \theta^2\sin(\theta)$

(g) $f'(x) = \dfrac{3x^2(3 - x) - (x^3 + 1)(-1)}{(3 - x)^2}$

(h) $f'(t) = \dfrac{-8t^{-3}}{(1 - 4t^{-2})^2}$

(i) $f'(x) = \dfrac{(x^2 - 4) - 2x^2}{(x^2 - 4)^2} - \dfrac{(x^2 + 4) - (x - 1)2x}{(x^2 + 4)^2}$

(j) $f'(y) = \dfrac{(\frac{1}{2}y^{-1/2} - \cos(y))(\tan(y) + y^5) - (\sqrt{y} - \sin(y))(\sec^2(y) + 5y^4)}{(\tan(y) + y^5)^2}$

(k) $f'(x) = \dfrac{(x + 3x^{-1}) - x(1 - 3x^{-2})}{(x + 3/x)^2}$

(l) $f'(x) = \dfrac{\left[2x\cot(x) + x^2(-\csc^2(x))\right](x + 1) - x^2\cot(x)}{(x + 1)^2}$

(m) $f'(x) = \sin(x) + x\cos(x) + \dfrac{\cos(x) + x\sin(x)}{\cos^2(x)}$

or $f'(x) = \sin(x) + \sec(x) + x\left[\cos(x) + \sec(x)\tan(x)\right]$

(n) $f'(x) = \dfrac{\frac{3}{2}x^{1/2}\sec(x)(2x + 3) - x^{3/2}\left[\sec(x)\tan(x)(2x + 3) + 2\sec(x)\right]}{\sec^2(x)(2x + 3)^2}$

(o) $f'(t) = \dfrac{2t(4t^2 - 12t + 9) - (t^2 + 1)(8t - 12)}{(4t^2 - 12t + 9)^2}$

3. Chain rule

(a) $f'(x) = \dfrac{1}{2}(x^2 - 3x^{-1} + 5)^{-1/2} \cdot (2x + 3x^{-2})$

(b) $f'(x) = 25(2\sin(x) - 3x^9)^{24} \cdot (2\cos(x) - 27x^8)$

(c) $f'(x) = -\csc(3x-1)\cot(3x-1) \cdot 3$

(d) $f'(r) = \pi \sec^2(\pi r^2 - 5r) \cdot (2\pi r - 5)$

(e) $f'(x) = 7\left(1 + (2x+1)^6\right)^6 \cdot 6(2x+1)^5 \cdot 2$

(f) $f'(x) = 3\sin^2(x^2)\cos(x^2) \cdot 2x$

(g) $f'(x) = \dfrac{1}{2}(\cos(x^2 - 3x^{-8} + 1))^{1/2}(-\sin(x^2 - 3x^{-8} + 1))(2x + 24x^{-9})$

(h) $f'(x) = \sec^2(\cot(x^2)) \cdot (-\csc^2(x^2)) \cdot 2x$

(i) $f'(x) = 2\sin(\cos^2(x)) \cdot \cos(\cos^2(x)) \cdot 2\cos(x)(-\sin(x))$

(j) $f'(x) = \sec(3x)\tan(3x)3\csc(5x) + \sec(3x)\left[-\csc(5x)\cot(5x) \cdot 5\right]$

(k) $f'(x) = 17(3x^2 - 9)^{16}(6x)(x^4 - x^2 + 1)^{31} + (3x^2 - 9)^{17} \cdot 31(x^4 - x^2 + 1)^{30}(4x^3 - 2x)$

(l) $f'(x) = \dfrac{\frac{1}{2}(x^2+1)^{-1/2} \cdot 2x \cdot (x^2-1)^{1/3} - (x^2+1)^{1/2} \cdot \frac{1}{3}(x^2-1)^{-2/3} \cdot 2x}{(x^2-1)^{2/3}}$

(m) $f'(x) = \dfrac{\cos(x^2) \cdot 2x\cos^2(x) + \sin(x^2) \cdot 2\cos(x)\sin(x)}{\cos^4(x)}$

(n) $f'(x) = 3x^2\tan^2(5x-3) + (x^3 - 8) \cdot 2\tan(5x-3)\sec^2(5x-3) \cdot 5$

(o) $f'(x) = \dfrac{4(3x^3+x)^3(9x^2+1)(\sqrt{x} - x^{-2})^5 - (3x^3+x)^4 \cdot 5(\sqrt{x} - x^{-2})^4(\frac{1}{2}x^{-1/2} + 2x^{-3})}{(\sqrt{x} - x^{-2})^{10}}$

(p) $f'(x) = 6\left(\dfrac{6x - x^4}{\sin(x)}\right)^5 \cdot \dfrac{(6 - 4x^3)\sin(x) - (6x - x^4)\cos(x)}{\sin^2(x)}$