201-103-RE - Calculus 1

WORKSHEET: DERIVATIVES (PART 2)

1. Find the derivative of each function, by using **rules for exponential and logarithmic functions**. DO NOT SIMPLIFY YOUR ANSWERS.

   (a) $f(x) = 3e^x - 7\log_{10}(x)$

   (b) $f(x) = \ln(x^2+1) - \log_2(5x)$

   (c) $f(x) = e^{x^2} \cdot \ln(\tan(x))$

   (d) $f(z) = \dfrac{2^z - z^2}{1 - \log_3(z)}$

   (e) $f(x) = e^{x\sec x} + e^5$

   (f) $f(x) = \dfrac{3^{-\sin(x)}}{1 + \ln(x^3 - x)}$

2. Find the derivative $y' = \frac{dy}{dx}$ in each case by using **implicit differentiation**. DO NOT SIMPLIFY YOUR ANSWERS.

   (a) $y^3 - y + \cos(x) + \sin(y) = x^{10}$

   (b) $x^2y + xy^2 = x^2 + 1$

   (c) $\dfrac{x^2 + y\cos(x) + 3}{y^2 - 1} = 5$

   (d) $ye^x + 2\log_2(x) - y^2 = \ln(x)$

   (e) $\sin(y^2) - \cos(xy) = 1$

   (f) $e^{x^2+y^2} = x^2 + y^2$

3. Find the derivative of the following functions, by using **logarithmic differentiation**. DO NOT SIMPLIFY YOUR ANSWERS.

   (a) $y = \ln\left(\dfrac{x^5}{(2x-1)^3(x^2+1)}\right)$

   (b) $y = \dfrac{(3x^3-1)^5}{x^4\tan^3(x)}$

   (c) $y = \dfrac{x^3(x+1)^4}{(2x+1)^5(3x+1)^6}$

   (d) $y = x^{(x^2)}$

   (e) $y = (x^4 + x)^{(2x-1)}$

   (f) $y = [\sin(x)]^{\cot(x)}$

4. Find the **equation of the tangent line** at the given point.

   (a) $f(x) = 5x\sin(x) + \dfrac{\pi}{2}$ at $x = -\pi/2$

   (b) $\ln(x+1) + e^y - x^2y = 1$ at the point $(0, 0)$

   (c) $(y-x)^3 + xy^3 - 27 = x$ at $x = 0$

5. Find where the tangent line is **horizontal** for each function.

   (a) $f(x) = 9x^4 - 40x^3 - 48x^2$

   (b) $f(x) = (2x+3)e^{x^2}$

   (c) $\sqrt{y} + x\log_2(x) - 4x = \cot(y) + \dfrac{x}{\ln(2)}$

6. Find the required **higher derivative** in each case.

   (a) find $f''(x)$ for $f(x) = \tan(x^3)$

   (b) find $f'''(x)$ for $f(x) = \ln(x^2+1)$

   (c) find $f^{(365)}(x)$ for $f(x) = \log_5(3x)$

ANSWERS:

1. Exponentials and Logarithms

   (a) $f'(x) = 3e^x - \dfrac{7}{x\ln(10)}$

   (b) $f'(x) = \dfrac{2x}{x^2+1} - \dfrac{5}{5x\ln(2)}$

   (c) $f'(x) = e^{x^2} \cdot 2x \cdot \ln(\tan(x)) + \dfrac{e^{x^2}\sec^2(x)}{\tan(x)}$

   (d) $f'(z) = \dfrac{(2^z\ln(2) - 2z)(1-\log_3(z)) - (2^z - z^2)\left(\frac{-1}{z\ln(3)}\right)}{(1-\log_3(z))^2}$

   (e) $f'(x) = e^{x\sec(x)}(\sec(x) + x\sec(x)\tan(x))$

   (f) $f'(x) = \dfrac{3^{-\sin(x)}(-\cos(x))\left[1+\ln(x^3-x)\right] - 3^{-\sin(x)}\left[\frac{3x^2-1}{x^3-x}\right]}{(1+\ln(x^3-x))^2}$

2. Implicit Differentiation

   (a) $y' = \dfrac{10x^9 + \sin(x)}{3y^2 - 1 + \cos(y)}$

   (b) $y' = \dfrac{2x - 2xy - y^2}{x^2 + 2xy}$

   (c) $y' = \dfrac{y\sin(x) - 2x}{\cos(x) - 10y}$

   (d) $y' = \dfrac{\frac{1}{x} - \frac{2}{x\ln(2)} - ye^x}{e^x - 2y}$

   (e) $y' = \dfrac{-y\sin(xy)}{2y\cos(y^2) + x\sin(xy)}$

   (f) $y' = \dfrac{2x - 2xe^{x^2+y^2}}{2ye^{x^2+y^2} - 2y}$

3. Logarithmic Differentiation

   (a) $y' = \dfrac{5}{x} - \dfrac{6}{2x-1} - \dfrac{2x}{x^2+1}$

   (b) $y' = \dfrac{(3x^3-1)^5}{x^4\tan^3(x)}\left[\dfrac{45x^2}{3x^3-1} - \dfrac{4}{x} - \dfrac{3\sec^2(x)}{\tan(x)}\right]$

   (c) $y' = \dfrac{x^3(x+1)^4}{(2x+1)^5(3x+1)^6}\left[\dfrac{3}{x} + \dfrac{4}{x+1} - \dfrac{10}{2x+1} - \dfrac{18}{3x+1}\right]$

   (d) $y' = x^{(x^2)}\left[2x\ln(x) + x\right]$

   (e) $y' = (x^4+x)^{(2x-1)}\left[2\ln(x^4+x) + (2x-1)\dfrac{(4x^3+1)}{x^4+x}\right]$

   (f) $y' = [\sin(x)]^{\cot(x)}\left[-\csc^2(x)\ln(\sin(x)) + \cot(x)\dfrac{\cos(x)}{\sin(x)}\right]$

4. Equation of Tangent Lines

   (a) $y = -5x + \frac{\pi}{2}$

   (b) $y = -x$

   (c) $y = \frac{1}{27}x + 3$

5. Horizontal Tangents

   (a) $x = -\frac{2}{3},\ 0,\ 4$

   (b) $x = -1,\ -\frac{1}{2}$

   (c) $x = 16$

6. Higher Derivatives

   (a) $f''(x) = 2\sec^2(x^3)\tan(x^3) \cdot 9x^4 + \sec^2(x^3) \cdot 6x$

   (b) $f'''(x) = \dfrac{-4x(x^2+1)^2 - (2-2x^2) \cdot 2(x^2+1) \cdot 2x}{(x^2+1)^4}$

   (c) $f^{(365)}(x) = \dfrac{364!}{x^{365}\ln(5)}$