201-103-RE - Calculus 1
WORKSHEET: OPTIMIZATION

EXAMPLE 1:

A total of x earthmovers will be sold if the price, in thousands of dollars is given by $p = 32 - \dfrac{x}{8}$. Find:

(1) an expression for the total revenue $R(x)$.
(2) the value of x that leads to maximum revenue.
(3) the maximum revenue.

EXAMPLE 2:

An investor has built a series of self-storage units near a group of apartment houses. He must decide on the monthly rental. From past experience, he feels that 200 units will be rented for \$15 per month with 5 additional rentals for each 25 cents reduction in the rental price. Let x be the number of 25 cents reductions in the price and find:

(1) an expression for the number of units rented.
(2) an expression for the price per unit.
(3) an expression for the total revenue.
(4) the value of x leading to maximum revenue.
(5) the maximum revenue.

PROBLEMS:

(1) A dress manufacturer has found that the number of dresses sold per month (x) , is related to the price in dollars per dress (p) by the equation $p = 500 - \frac{x^2}{300}$. Determine whether total revenue is increasing or decreasing when $x = 100,\ 200,\ 300$.

(2) A company manufacturing bathing suits sells the suits at \$24 each. The dollar cost of producing x suits is $C(x) = 150 + \frac{39}{10}\,x + \frac{3}{1000}\,x^2$. Express profit as a function of x and determine the number of bathing suits the company should produce to achieve maximum profit.

(3) Boxed greeting cards cost the distributor 60 cents/box. He can sell 100 boxes at \$1/box. For each cent the price is lowered he can increase the number of boxes sold by 5.
   (a) If x is the number of boxes sold, show that the total revenue in cents is $R(x) = \frac{1}{5}\left(600x - x^2\right)$.
   (b) Define the profit function, $P(x)$.
   (c) How many boxes should he sell to maximize profit ?
   What would the price be then ? What would the profit be in \$?

(4) A restaurant chain is planning a new dining room. Estimates of monthly profit per chair, indicate that for 100 chairs the profit/chair would be \$24. If seating capacity is over 100 chairs, the monthly profit/chair decreases by 10 cents per chair added. What seating capacity will maximize profit ?

(5) A Florida orange grower finds that the average yield/tree is 400 oranges if no more than 16 trees are planted in each plot. For each additional tree per plot the yield of each tree is decreased by 20 oranges. How many trees should be planted in each plot to maximize yield?

(6) An all news radio station has made a study of the listening habits of local residents between 5 p.m. and midnight . The results indicate that x hours after 5 p.m. on a typical week night: $-\frac{x^3}{4} + \frac{27}{8}\,x^2 - \frac{27}{2}\,x + 30$ percent of the population is tuned into the station.
   (a) At what time between 5 p.m. and midnight are there most listeners ?
   (b) At what time between 5 p.m. and midnight are there fewest listeners ?

(7) Plans for a drugstore require 14,400 square feet of floor space. The floor will be rectangular with 3 brick walls and an all glass front.
   (a) If glass costs 1.88 times as much as brick (brick costs \$50 a linear foot), which dimensions of floor would minimize cost of materials ?
   (b) If heat loss is 7 times as great through glass as through brick (heat loss through brick is 50 kcal a linear foot), which dimensions of floor would minimize heat loss ?

(8) The daily production cost for a factory to manufacture x deluxe contour chairs is given to be $\$\left(500 + 14x + \frac{x^2}{2}\right)$. The price function is $\$\left(150 - \frac{3}{2}\,x\right)$.
   (a) Write the equation of the revenue function, $R(x)$.
   (b) Write the equation of the profit function, $P(x)$.
   (c) Evaluate the marginal cost for $x = 10,\ x = 30,\ x = 50$.
   Evaluate the marginal profit for $x = 10,\ x = 30,\ x = 50$.
   (d) How many chairs should be produced daily to maximize the profit ?

(9) $\$\left(\frac{x^2}{3} + 34x + 40\right)$ is the total cost of production per day for x Cassette players and they are sold for $\$\left(62 - \frac{x}{4}\right)$ per Cassette player. What daily production will produce a maximum profit ?

(10) A stereo manufacturer determines that in order to sell x units of a new stereo, its price per unit must be $p = 1000 - x$. It also determines that the total cost of producing x units is given by $C(x) = 3000 - 20x$.
   (a) Find the total revenue equation, $R(x)$.
   (b) Find the total profit equation, $P(x)$.
   (c) How many units must the company produce and sell to maximize profit ?
   (d) What is the maximum profit?
   (e) What price per unit must be charged to make maximum profit?

(11) From a thin piece of cardboard, 8 in. by 8 in., square corners are cut out so that the sides can be folded up to make a box. What dimensions will yield a box of maximum volume ? What is the maximum volume ?

(12) Of all numbers whose sum is 50, find the two which have maximum product. Can there be a minimum product ? Explain.

(13) Suppose you are the owner of a 95 unit motel. All units are occupied when you charge \$45 a day per unit. For every increase of x dollars in the daily rate, there are x units vacant. Each occupied room costs \$20 a day to service and maintain. What should you charge per unit to maximize profit ?

(14) A truck burns fuel at the rate of $G(x) = \frac{1}{30}\left(\frac{8100}{x} + x\right)$ litres per km when travelling x km per hour on a straight level road. If fuel costs 60 cents per litre, find the speed that will produce the minimum total cost for a 1000-km trip.

(15) A railroad company offers the following discount rates for chartered trips of at least 400 passengers; the fare is \$40 per person for exactly 400 passengers, while the fare per person decreases by five cent for each passenger over 400. Find the number of passengers which will yield the maximum revenue. What is the maximum revenue?

(16) The National Forest Service finds that 16 trees planted on an acre of land will grow approximately 3 feet per year. For each additional tree planted, the growth will be reduced by 0.5 inch. Find the number of trees per acre that will yield the largest amount of timber (wood).

(17) A group of students arrange a chartered flight to Paris. The charge per student is \$800 if 100 students go on the flight. If more than 100 participate, the charge per student is reduced by an amount equal to \$4 times the number of students above 100.
 (a) Find the total revenue equation, R, as a function of x (the number of students above 100).
 (b) Find the number of students that will provide maximum revenue.
 (c) What is the charge per student when maximum revenue is obtained.

(18) A school wants to enclose behind a wall of the building 2 adjacent equal rectangular courtyards of total area 19200 square feet with a fence that costs \$50 per foot. Find the dimensions of each courtyard to minimize the cost of the fence. Find the minimum cost.

(19) A sports complex wants to enclose 3 adjacent equal tennis courts with a fence that costs \$20 per linear meter. The budget of the fence is \$12 000 ; find the dimensions of each tennis court to maximize the area. Find the maximum area.

(20) A farmer wants to enclose 2 adjacent equal rectangular fields of total area 12000 square meters. The exterior fence costs \$50 per meter and the interior fence costs \$20 per meter. Find the dimensions of each field to minimize the cost of the fence. Find the minimum cost.

ANSWERS:

(1) $R'(100) > 0\,;\ R'(200) > 0\,;\ R'(300) < 0$

(2) $P(x) = -\dfrac{3}{1000}\,x^2 + \dfrac{201}{10}\,x - 150$;to maximize profit: 3350 Bathing suits.

(3 b) $P(x) = \dfrac{3}{50}\,x - \dfrac{x^2}{500}$ dollars

(3 c) 150 boxes ; Price/unit: 90 cents ; Max $P(150) = \$45$

(4) 170 chairs (5) 18 trees/plot

(6 a) Maximum: Most listeners at 5:00 P.M. (6 b) Minimum: Fewest listeners at 8:00 P.M.

(7 a) 100 ft by 144 ft ; (7 b) 60 ft by 240 ft

(8 a) $R(x) = 150x - \dfrac{3}{2}\,x^2$ (8 b) $P(x) = -2x^2 + 136x - 500$

(8 c) marginal cost: $C'(x) = 14 + x \implies C'(10) = 24\,,\ C'(30) = 44\,,\ C'(50) = 64$ \$/chair
marginal profit: $P'(x) = -4x + 136 \implies P'(10) = 96\,,\ P'(30) = 16\,,\ P'(50) = -64$ \$/chair (8 d) 34 chairs

(9) 24 cassette players

(10 a) $R(x) = 1000x - x^2$ (10 b) $P(x) = -x^2 + 1020x - 3000$ (10 c) 510 units
(10 d) Maximum profit: $P(510) = \$257,100$ (10 e) Price per unit: $p = \$490$

(11) dimensions of the box are 16/3 inches by 16/3 inches by 4/3 inches with maximum volume of the box is about 37.9 cubic inches

(12) the 2 numbers are 25 and 25 (13) \$80 (14) speed is 90 km/h

(15) 600 passengers ; maximum revenue is \$18000 (16) 44 trees/acre

(17 a) $R(x) = 80000 + 400x - 4x^2$ (17 b) 150 students (17 c) Charge/student: \$600

(18) dimensions are 80 by 120 ft each courtyard ; minimum cost is \$24000

(19) dimensions are 50 by 75 m each tennis court ; maximum area is 11250 square meters

(20) dimensions of each field are 60 by 100 ft ; minimum cost is \$24000