

# Mathematical Puzzles, Games and Other Diversions

Day 4

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# Counting

Suppose you're eating at a restaurant where you must choose an appetizer, a main course and a dessert.

- ▶ For an appetizer, you can choose a tomato soup or a chef salad.
- ▶ For the main course, your options are roast beef, grilled chicken or tofu stew.
- ▶ For dessert, they have ice cream and cake.

How many possible different meal combinations can you order?

## Counting (cont.)

- ▶ What is the probability that a card drawn from a shuffled deck of 52 cards is an ace?
- ▶ What is the probability that two cards drawn are both aces?
- ▶ And three cards, or four cards?
- ▶ How many different ways are there to order the four aces?
- ▶ How many different orders are there for a deck of cards?
- ▶ What is the probability that if you draw three cards from a deck, at least one of them is an ace?
- ▶ What is the probability that if you draw three cards from a deck, exactly two of them are aces?

## More Counting

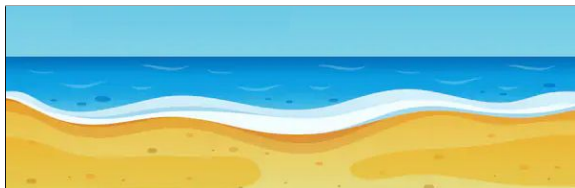
- ▶ How many different ways can I form a 3-person group in a class of 10 students?
- ▶ Given a race with 10 people, how many ways are there to assign the gold, silver and bronze medals (no ties)?

When order matters, we're dealing with permutations, when it doesn't, we're using combinations.

- ▶ What is the probability that if draw two cards from a deck of cards, the FIRST one you draw is an ace?
- ▶ What is the probability that if you draw two cards from a deck of cards, the SECOND one you draw is an ace?
- ▶ What is the probability that if you draw two cards from a deck of cards, that at least one of them is an ace?

## A basic example of Game Theory

(Hotelling's Law) There's a 1 km stretch of beach where Alice plans to set up an ice cream shop.



- ▶ Assuming an even distribution of people, where should Alice set up shop?
- ▶ Suppose Bob also wants to set up an ice cream shop. Where should they set up their shops to best suit the beachgoers?
- ▶ What can Alice (or Bob) do to get more customers?

Where else have you seen this kind of situation occur?