

# Mathematical Puzzles, Games and Other Diversions

Day 5

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# Problem Solving

From *How To Solve It* (1945) by George Polya



G. PÓLYA, 1887–1985

## Four Principles

1. Understand the Problem
2. Devise a Plan
3. Carry Out The Plan
4. Review (and Generalize)

# Problem Solving (cont.)

## 1. Understand the Problem

This seems so obvious, but it's not. And if you don't really understand the problem, there's no way you can answer it properly.

### **Some Good Questions**

- ▶ Do you understand all the words and notation?
- ▶ What's the goal? What are you being asked?
- ▶ Can you restate the problem in your own words?
- ▶ Would drawing a picture or diagram help your understanding?
- ▶ Is there enough information to enable you to find a solution?

Don't overlook this step. You do so at your own peril.

# Problem Solving (cont.)

## 2. Devise a Plan

Now figure out how you're going to attack the problem. Choosing an appropriate strategy gets easier with thoughtful practice.

### Some Examples

- ▶ Guess and Check
- ▶ Make an ordered list
- ▶ Use symmetry
- ▶ Solve a simpler version
- ▶ Look for a pattern
- ▶ Draw a picture/diagram
- ▶ Solve an Equation
- ▶ Work backward
- ▶ Use a formula
- ▶ Be Creative

This is hard work, so be patient with yourself.  
Ideas often need to simmer.

# Problem Solving (cont.)

## 3. Carry Out the Plan

Following through on your plan is the easy part, if you've got the skills. It will just take some care and patience.

And if your plan isn't working out, go back and make a new plan. It's normal that plans don't work out. Just keep at it.

## 4. Review (and Generalize)

- ▶ Check your work to make sure you've solved the problem.
- ▶ Reflect on what you've done, what worked and what didn't.
- ▶ Think about other problems where your ideas could be useful.

This step is vital for you to get better at solving problems in the future. Don't ignore it.

# Problem Solving (cont.)

## Addendum

“If you cannot solve the proposed problem, try to solve first some related problem. Could you imagine a more accessible related problem?”

## Final Considerations

- ▶ You should be considering these principles when you're solving any math problem.
- ▶ Great mathematicians have followed Polya's guidance to make important discoveries.
- ▶ The general principles apply beyond just math.

## Project Ideas

- ▶ Fitch-Cheney Poker Trick and Variations
- ▶ Topological oddities (mobius strips, klein bottles, knot theory)
- ▶ Blackjack and Card Counting
- ▶ Speed Math (fast multiplication, roots, day of the week)
- ▶ Rubik's Cube or other mathematical toys
- ▶ Sudoku
- ▶ Pentominoes
- ▶ Conway's Game of Life
- ▶ Counterfeit Coin Problems
- ▶ Shuffling Cards (e.g. perfect shuffles, randomization, tracking, patterns)
- ▶ Cryptography (coding/decoding messages)

## Project Ideas (cont.)

- ▶ Prime Numbers
- ▶ Game Theory
- ▶ Electoral Systems and fairness criteria
- ▶ Gerrymandering and mathematical solutions
- ▶ Queueing (math of traffic flow, lines at the grocery store)
- ▶ Nim and variant games
- ▶ Menace/AI (tic-tac-toe playing AI built using matchboxes)
- ▶ Special Numbers: (e.g. Pi, e or the golden ratio)
- ▶ Infinity
- ▶ Surprising results in prob/stats (Base rate fallacy, simpson's paradox)
- ▶ Mathematically Stacked Decks
- ▶ Graph Theory (Euler tours, Hamiltonian circuits)
- ▶ Mathematical Fallacies



## More Project Ideas

- ▶ Writing a program that solves a puzzle, plays a game, performs a magic trick.
- ▶ Present and explain a reasonably difficult mathematical riddle/puzzle/trick
- ▶ Profile of a mathematician and his/her work (e.g. Martin Gardner, Ada Lovelace, Lewis Carroll, Alan Turing)
- ▶ Profile and explanation how somebody beat the casino, the lottery or horse racing.
- ▶ Collecting and/or analysing specific data (baseball stats, gambling systems, population growth, etc)
- ▶ Collect/present rules and strategy for mathematical game(s).